DESIGN OF SELF – SUPPORTING DOME ROOFS

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INTRODUCTION

The domes on the circular-shaped base have a large usage – silos, tanks, warehouses for bulk materials, sportive facilities and exhibition halls. They are light, beautiful and can cover big spans, providing free space without intermediate columns. The steel domes are successfully applied on the spans with diameter $\boldsymbol{D} \leq 50{,}0$ m.
A major part of the domes are used to cover steel tanks for oil storage. Traditionally their construction is composed by radial elements (girders), ring-shaped elements (rings) and roof cover plates (Fig. 1).

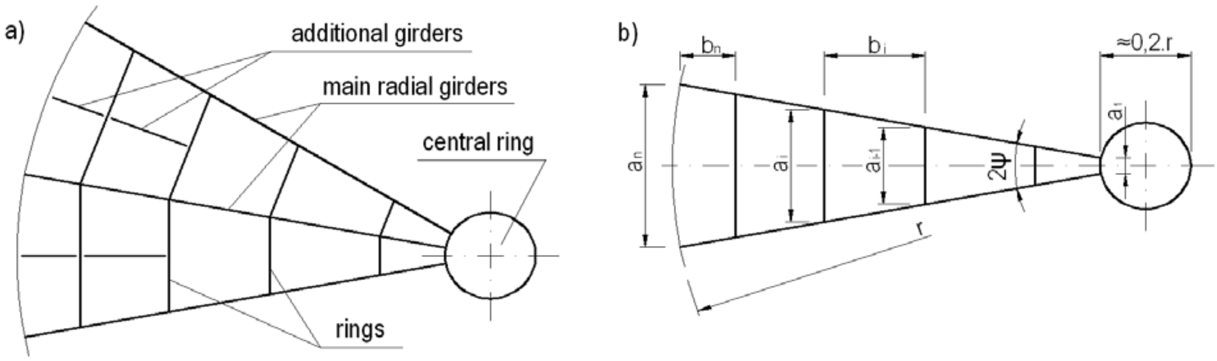

Fig. 1 Structure of the self-supporting roof
a) basic elements b) design scheme

1 LOADS AND LOAD COMBINATIONS

The calculation of roof domes during the exploitation usually is done for two load combinations. In the first of them the summarized loading $\boldsymbol{q_1}$ operates from top to the bottom. In the second one the loading on the roof $\boldsymbol{q_2}$ operates from the bottom to the top:

$$q_1 = \max\begin{cases} \gamma_{\mathrm{Fg,sup}} . g_{\mathrm{n}} + \gamma_{\mathrm{Ft}} . g_{\mathrm{tn}} + \gamma_{\mathrm{Fs}} . s_{\mathrm{n}} + \gamma_{\mathrm{Fv}} . \psi_0 . p_v^n \downarrow \\ \gamma_{\mathrm{Fg,sup}} . g_{\mathrm{n}} + \gamma_{\mathrm{Ft}} . g_{\mathrm{tn}} + \gamma_{\mathrm{Fv}} . p_v^n + \gamma_{\mathrm{Fs}} . \psi_0 . s_{\mathrm{n}} \downarrow \end{cases} \tag{1.1}$$

$$q_2 = \max\begin{cases} \gamma_{\mathrm{Fa}} . p_{\mathrm{o}}^{\mathrm{n}} + \gamma_{\mathrm{Fw}} . \psi_0 . w_n - \gamma_{\mathrm{Fg,inf}} . (g_n + g_{\mathrm{tn}}) \uparrow \\ \gamma_{\mathrm{Fw}} . w_n + \gamma_{\mathrm{Fa}} . \psi_0 . p_{\mathrm{o}}^{\mathrm{n}} - \gamma_{\mathrm{Fg,inf}} . (g_n + g_{\mathrm{tn}}) \uparrow \end{cases} \tag{1.2}$$

where g_{n} is a characteristic values of dead loads (permanent actions);
g_{tn} is characteristic value of weight of heat insulation on the roof (if any);
p_v^n is characteristic value of negative internal pressure under dome;
s_{n} is characteristic value of snow loading on the roof;
p_o^n is characteristic value of internal pressure under the dome;
w_n is characteristic value of wind load on the dome;
$\gamma_{\mathrm{Fg,sup}}$, $\gamma_{\mathrm{Fg,inf}}$, γ_{Fa}, γ_{Fv}, γ_{Fs}, γ_{Fw} – partial factors for actions, according to EN 1990 [3];
ψ_0 is factor for combination value of a variable action [3].

2 FORCES IN THE DOME'S ELEMENTS

Spatial steel domes with radial girders and circular elements are many times undetermined spatial systems, which forces into elements difficultly can be calculated through manual solution. The precise calculation of forces in elements of steel structure of dome is done with use of suitable software, considering the stiffness of the elements and flexibility of the joints.

Preliminary design of the elements of the steel structure of dome roofs could be done to obtain the cross section and type of the elements that are defined in the FEA model. Here will be shown two approaches to receive these steel sections.

2.1 Forces in the elements of dome, according to procedures of E. Лессиг [2]

The shown there methodology is used from Bulgarian engineers for a long time. Of course, it is applied for a lot of dome roofs of steel tanks that are in service now.

Calculations of the section of elements of spherical domes with radial girders and circular elements are done on the base of pin - joint system. Characteristic feature of the methodology is that effective length of compressed elements is equal to real geometrical length (distance between pin joints) .

The biggest axial force $\boldsymbol{S}_\mathbf{i}$ (pressure in combination q_1 and tension in q_2) in the dome's radial girders is done when a full design load operates on its whole surface. The efforts in the girder immediately upon the ***i***-th joint are calculated according to the formula:

$$S_i = \frac{A_i.q}{n.\sin\alpha_\mathrm{i}} \tag{2.1}$$

where A_i is a surface of circle, limited by the ***i***-th ring;

q is loading of combination q_1 or q_2;

n is number of main radial girders in the dome;

α_i is angle between the tangent to the girder in the ***i***-th joint and horizontal plane.

Except axial forces in the girders there are additional bending moments caused by distributed loads in the fields, limited by the steel structure. (Fig. 2).

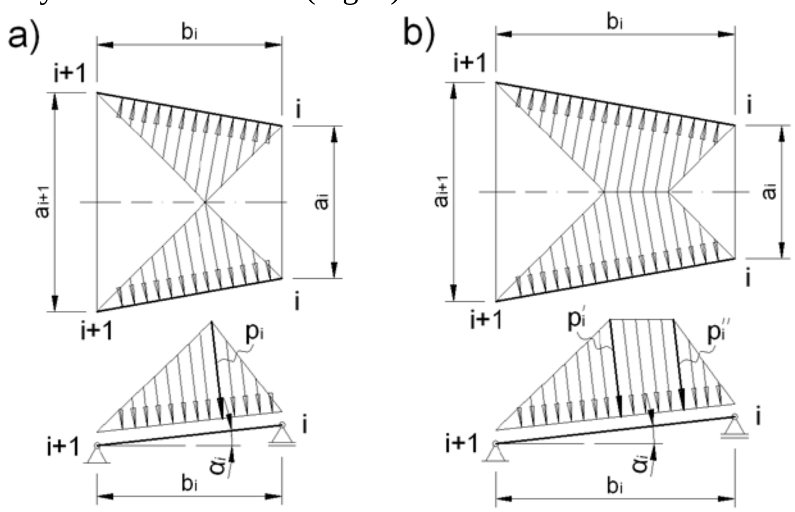

Fig. 2 Scheme for girders loading

a) with triangle shaped load b) with trapezium shaped load

When distance $\bar{a}_\mathrm{i} \geq b_\mathrm{i}$, on the girder accepted as a simply supported beam with opening $\boldsymbol{b}_\mathbf{i}$, operates a load distributed according to the triangle low. (Fig. 2 – a). The maximum bending moment $\boldsymbol{M}_\mathbf{max}$ would be calculated as in the simply supported beam, with the expression:

$$M_{\max} = \frac{1}{12} p_\mathrm{i}.b_\mathrm{i}^2 \tag{2.2}$$

When distance $\bar{a}_\mathrm{i} < b_\mathrm{i}$, the distributed load has a trapezium shaped outline (Fig. 2 – b) and the value of the of maximum bending moment $\boldsymbol{M}_\mathbf{max}$ is:

$$M_{\max} = \frac{1}{24} p_i.\left(3.b_i^2 - \overline{a}_i^2\right) \tag{2.3}$$

where $\overline{a}_i$ is average width of field.

The value of $\boldsymbol{p}_\mathbf{i}$ in the formulas (2.2) and (2.3) is calculated by summing up of the loadings of two neighbouring fields according to the schemes shown on Fig. 2

In the vertical plane main roof girders are measured as a compressed with a bending moment (when the load is $\boldsymbol{q}_\mathbf{1}$**)** and/or tensioned with a bending moment (when the load is $\boldsymbol{q}_\mathbf{2}$).

When there is a steel sheets welded to the construction the stability of the girders in horizontal plane is assured.

When steel cover plates are not a welded to the construction and roof inclination is i > 1:16, the radial girders have to be checked for general loss of stability between joints of radial girders and circular rings. It is accepted that their effective length is equal to the geometrical.

The maximum of compression forces $\boldsymbol{T}_\mathbf{c,i}$ in the ***i***-th joint, appeared when there is a snow loading out of the circle of exanimated ring (Fig. 3). Its calculation is according to the formula:

$$T_{c,i} = \frac{1}{2.n.\sin\psi}.\left(\cot g(\alpha_i).q_1^{'}.A_i - \cot g(\alpha_{i+1}).\left(q_1.\left(A_{i+1} - A_i\right) + q_1^{'}.A_i\right)\right) \tag{2.4}$$

where:

$$q_1^{'} = \gamma_{Fg,sup}.g_n + \gamma_{Ft}.g_{tn} + \gamma_{Fv}.p_v^n \downarrow \tag{2.5}$$

$$2.\psi = \frac{2.\pi}{n}, \text{ [rad]} \tag{2.6}$$

in which $\boldsymbol{n}$ is the number of the main radial girders (Fig. 1).

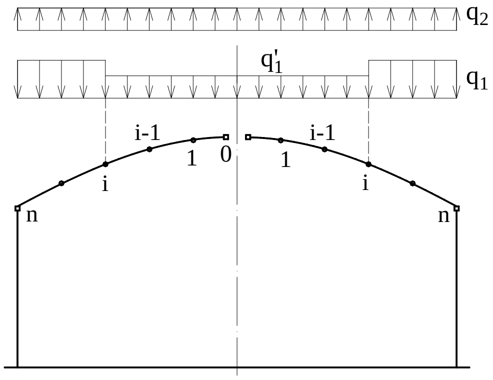

Fig. 3 Calculation of the forces in the intermediate rings.

The maximum tension forces $\boldsymbol{T}_\mathbf{t,i}$ in the ***i***-th joint is calculated as follow:

$$T_{t,i} = \frac{q_2}{2.n.\sin\psi}.\left(\cot g(\alpha_{i+1}).A_{i+1} - \cot g(\alpha_i).A_i\right) \tag{2.7}$$

Bending moments appear in ring's elements as a result of roof loading. For its determination the ring's elements are accepted as simply supported beams with length equal to distance between joints.

Elements of the rings are measured to be able to bear compression forces with bending moments or tension forces with a bending moments, depending on loading on them (combination $\boldsymbol{q}_1$ or $\boldsymbol{q}_2$). It is accepted that their effective length is equal to the geometrical. Maximum bending moments are calculated on formulas, analogical on (2.2) и (2.3).

The sections of the steel construction of the dome calculated through described above analytical methodology are close to the really necessary [1]. Often the sections calculated by this way are safer but it is not recommendable to calculate them only analytically. FEA solutions give the possibility to improve the steel structure and to account influence of different asymmetric loads on the domes.

2.2 Forces in the elements of dome, according to procedures of EN 1993-4-2 [5]

In this methodology, written in European standard EN 1993-4-2, number, sections and positions of steel rings are not taken into consideration. Whole attention is focused on radial girders. There is not accounted simultaneously work of radial girders and rings as a spatial frame structure there. On that reason this methodology is appropriate for dome roofs where a steel structure is above roof cover plates.

To use written methodology in that standard should be done:

- diameter of the tank is less than 60,0 m;
- distributed load does not deviate strongly from symmetry about the tank axis.

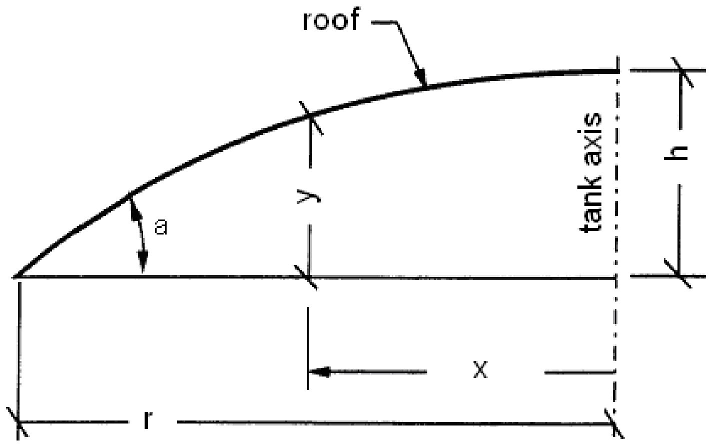

Fig. 4 Coordinates of dome roof

For spherical roofs under the action of distributed loads, the maximum vertical design force per radial girder should be taken as:

$$P_{\mathrm{Ed}} = \beta . r^2 p_{\mathrm{v,Ed}} \tag{2.8}$$

in which r is the radius of the tank (Fig. 4);

$p_{\mathrm{v,Ed}} = q_1$ - maximum vertical component of the design distributed load

$$\beta = \frac{\pi}{n}, \mathrm{rad} \tag{2.9}$$

where n is number of main radial girders

The normal force N_{Ed} and bending moment M_{Ed} in each rafter for design according to EN 1993-1-1 [4] may be obtained from:

$$N_{\mathrm{Ed}} = 0{,}375 \frac{r}{h} P_{\mathrm{Ed}} \tag{2.10}$$

$$M_{\mathrm{Ed}} = \frac{1}{3}\left(\frac{r}{1-\varepsilon}\right)\left\{1-\left(\frac{x}{r}\right)^3 - 1{,}10\left(\frac{y}{h}\right)\right\} P_{\mathrm{Ed}} \tag{2.11}$$

provided that the following conditions are met:

$$p_{\mathrm{v,Ed}} \geq 1{,}2\ \mathrm{kN/m^2}$$

$$I_{\mathrm{y}} \geq \frac{N_{\mathrm{Ed}} . r^2}{\pi^2 . E} \tag{2.12}$$

$$b_{\mathrm{K}} \geq 2.h_{\mathrm{K}} \tag{2.13}$$

$$A_1 \geq A_2 \tag{2.14}$$

$$h_{\mathrm{K}}^2 \left(\frac{A_1 . A_2}{A_1 + A_2}\right) \geq \frac{I_{\mathrm{y}}}{2.\beta} \tag{2.15}$$

in which:

$$\varepsilon = N_{\mathrm{Ed}} \frac{(0{,}6r)^2}{\pi^2 E I_y} \tag{2.16}$$

where h is the rise of the tank roof (Fig. 4);
x is the radial distance from the centreline of the tank;
y is the vertical height of the roof at coordinate x;
b_K is the flange width of centre ring;
h_K is vertical distance between flanges of the centre ring;
A_1 is the area of top flange of the centre ring;
A_2 is the area of bottom flange of the centre ring;
I_y is the second moment of area of the rafter about the horizontal axis.

3 ANALYTICAL RESEARCH OF SPATIAL STEEL DOMES

Several steel domes with circular base are examined. They have different heights of over passing and diameters ***D*** of the base. They are situated in different points of the Earth Globe and the loads that impact them are not the same. Steel plates are not welded to the roof's structure so we will research only on the load combination $\boldsymbol{q_1}$.
The examined in the current research domes are designed in the real life. A part of them are constructed and the rest of them are under construction now.
Firstly, the examined domes are calculated analytically through the described in point 2.1 methodology. Based on the calculated forces $\boldsymbol{S_i}$, $\boldsymbol{T_i}$ and $\boldsymbol{M_{max}}$, and using the methodology in Eurocode 1993-1-1 [4], are calculated steel hot rolled sections which can bear them.
After that, examined domes are calculated analytically through the described in point 2.2 methodology. Based on the calculated forces $\boldsymbol{N_{Ed}}$ and $\boldsymbol{M_{Ed}}$, using the methodology in Eurocode 1993-1-1, are calculated steel hot rolled sections.
In the tables below is shown a part of examined in the current research domes with their diameters ***D*** in the base. Roof cover plates are above the steel structure.
Radial girders of dome roof are checked for:

- mounting conditions - the same for both methodologies;
- working conditions.

Steel sections, received as a result of calculations with these 2 methodologies are compared.

Table 1 Tank T080, La Reunion, France

Volume V = 3000 m^3	Diameter D = 16 m	Height of tank H = 15 m
Radius of curvature of dome roof – R_r = 20,5 m		
Methodology of E. Лессиг [2]		
radial girders - IPE100, steel S235, 24 pcs.	M_{Ed} = 2,35 kN.m	N_{Ed} = -56,46 kN
rings – IPE 80, steel S235, 4 pcs.		
Weight of roof structure – 2 277 kg		
Methodology of EN 1993-4-2 [5]		
radial girders - IPE140, steel S235, 24 pcs.	M_{Ed} = -6,82 kN.m	N_{Ed} = -47,06 kN
rings - IPE 80, steel S235, 4 pcs., put to support cover plates and to stabilize radial girders		
Weight of roof structure – 3 131 kg		

Table 2 Tank T016, Martinique, France

Volume V = 14 650 m^3	Diameter D = 36 m	Height of tank H = 15,6 m
Radius of curvature of dome roof – R_r = 48 m		
Methodology of E. Лессиг [2]		
radial girders – IPE220, steel S235, 48 pcs.	M_{Ed} = 4,36 kN.m	N_{Ed} = -173,72 kN
rings – IPE 140, steel S235, 7 pcs.		
Weight of roof structure – 28 058 kg		

Methodology of EN 1993–4-2 [5]		
radial girders – IPE270, steel S235, 48 pcs.	M_{Ed} = -41,55 kN.m	N_{Ed} = -125,53 kN
rings - IPE 80, steel S235, 7 pcs., put to support cover plates and to stabilize radial girders		
Weight of roof structure – 32 614 kg		

Table 3 Tank T111, Braakmanhaven, Netherlands

Volume V = 40 000 m^3	Diameter D = 52,5 m	Height of tank H = 20 m
Radius of curvature of dome roof – R_r = 71,40 m		
Methodology of E. Лессиг [2]		
radial girders – IPE240, steel S355, 72 pcs.	M_{Ed} = 2,92 kN.m	N_{Ed} = -281,45 kN
rings – IPE 180, steel S355, 12 pcs.		
Weight of roof structure – 74 807 kg		
Methodology of EN 1993–4-2 [5]		
radial girders – IPE360, steel S355, 72 pcs.	M_{Ed} = -98,53 kN.m	N_{Ed} = -203,70 kN
rings - IPE 80, steel S235, 12 pcs., put to support cover plates and to stabilize radial girders		
Weight of roof structure – 107 657 kg		

There could be noted that received according to first methodology [2] steel sections are checked using appropriate structural software and 3D models.

For the needs of the FEA a number of independent solutions are provided, using different initial conditions. They are:

- geometrically linear solution without taking into account large deformations;
- geometrically nonlinear solution with taking into account P-D effects and large deformations
- analysis of the structure for loosing stability with taking into account the geometric imperfections, which are included in the model as a shifting 1/100 of spans.

For distributed loads that do not deviate strongly from symmetry about the tank axis, preliminary accounted according to [2] steel sections are fully adequate.

4 CONCLUSIONS

Precise calculation of forces in elements of dome roofs could be done only with use of suitable software, considering the stiffness of the elements and flexibility of the joints. Usually, preliminary analytical design is done to obtain the cross section and type of the elements that are defined in the FEA model. Various analytical methodologies could be used. One of them is written in [2] more than 40 years ago, well known in Bulgaria. Other is shown in European standard EN 1993–4–2:2007 [5]. These 2 methodologies have different conceptions, equations and, of course, results.

Many investors want methodology in EN 1993–4–2:2007 [5] to be followed step by step, blind. In case of dome roofs which have radial girders and circular rings, structural elements work spatial, together, supporting each other. Unfortunately, methodology of [5] does not account it. As a result, we receive weak rings and heavy radial girders, but sum of their weights is bigger than received according to [2] and FEA.

In that reason we propose to be written in European standard EN 1993–4–2:2007 [5] that shown methodology for design is applicable for dome roofs with radial girders only. For dome roofs with radial girders and circular elements should be used different, more appropriate methodology.

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