LATERAL – TORSIONAL BUCKLING OF STEEL BEAMS

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Abstract: *Beams are structural elements subjected to bending loads transverse to their longitudinal axis. For steel beams, which compressed flange is not laterally restrained, checking loss of overall stability is often authoritative in determining their section.*
In engineering practice are known various approaches to verify the assurance of the steel beam against lateral-torsional buckling. In this article the attention is focused to the methods and their characteristics, described in actual version of the European standard EN1993-1-1.

Key words: *steel beams, lateral-torsional buckling, critical bending moment, FEA*

Beams are structural elements which are bended by different loads, transverse to their longitudinal axis. Checking the loss of overall stability often is very important for determination of the section of steel beams which compressed flange is not laterally restrained.

In engineering practice are known various approaches to verify the assurance of the steel beam against lateral-torsional buckling. This article is focused on various methods and their characteristics, described in actual version of European standard EN1993-1-1 [2].

1. Bearing capacity of steel beams in the condition of stability loss. General case.

According to the current version of the standard EN 1993-1-1 [2], bended about "strong" axis "y-y" steel element, which compressed flange is not laterally restrained, should be checked for buckling according to the following criteria:

(1.1) $$M_{y,Ed} \le M_{b,Rd} \,,$$

where:
$M_{y,Ed}$ is the design value of bending moment by axis "y-y ";
$M_{b,Rd}$ – design buckling resistance moment. It is determined according to formulae:

(1.2) $$M_{b,Rd} = \chi_{LT} . W_y . \frac{f_y}{\gamma_{M1}} \,,$$

where:
χ_{LT} is a reduction factor for lateral - torsional buckling;
f_y – characteristic value of yield strength of steel;
γ_{M1} – coefficient for bearing capacity of elements in the condition of loss of stability;
W_y - section modulus about axis "y-y":

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$W_y = W_{pl,y}$ is a correspondent to the compressed flange section modulus, when the section of the beam is class 1 or 2;

$= W_{el,y}$ is a correspondent to the compressed flange elastic section modulus, when the section of the beam is class 3;

$= W_{eff,y}$ is a correspondent to the compressed flange effective section modulus, when the section of steel beam is class 4.

For bended elements with constant sections, the reduction factor χ_{LT} for the appropriate non-dimensional slenderness $\bar{\lambda}_{LT}$, should be determined from:

$$\text{(1.3)} \quad \chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \bar{\lambda}_{LT}^2}} \leq 1{,}0,$$

where:

$$\text{(1.4)} \quad \Phi_{LT} = 0{,}5.\left[1 + \alpha_{LT}.\left(\bar{\lambda}_{LT} - 0{,}2\right) + \bar{\lambda}_{LT}^2\right],$$

in which:

α_{LT} is a coefficient for imperfections, determined by the standard EN1993-1-1 [2];

M_{cr} – elastic critical moment for lateral - torsional buckling. It is based on gross cross sectional properties and takes into accounts loading conditions.

$$\text{(1.5)} \quad \bar{\lambda}_{LT} = \sqrt{\frac{W_y.f_y}{M_{cr}}}$$

In the final version of the EN1993-1-1 by 2005, analytical methodology for determining the elastic critical moment M_{cr} is removed. The document recommends its determining to be done by established methods. For example - using FEA.

2. Simplified methods for beams reinforcing building

As determining M_{cr} is obviously heavy and long process , standard EN1993-1-1 gives an opportunity to check the general loss of stability using simplified methods. According to these methods there is no need to check the beams for lateral - torsional buckling when value $\bar{\lambda}_f$ of the relevant compressed flange correspond to the condition:

$$\text{(2.1)} \quad \bar{\lambda}_f = \frac{k_c.L_c}{i_{f,z}.\lambda_1} \leq \bar{\lambda}_{c,0}.\frac{M_{c,Rd}}{M_{y,Ed}},$$

where:

k_c is a correction factor , reporting the distribution of the moment between the lateral restraints;

L_c – length of the compressed flange of the beam between the lateral restraints;

$i_{f,z}$ - radius of inertia about "weak" axis (axis "z-z") of equivalent compressed element, composed by compressed flange and 1/3 of the compressed part of web;

$\bar{\lambda}_{c,0} = 0{,}5$ – border value of the equivalent compressed flange;

$M_{y,Ed}$ – maximum design value of the bending moment about axis "y-y", in laterally restrained part of element;

$M_{c,Rd}$ – design resistance moment of steel section, bending about axis "y-y". It is calculated according to the formulae:

$$(2.2)\quad M_{c,Rd} = W_y . \frac{f_y}{\gamma_{M1}},$$

$$(2.3)\quad \lambda_1 = \pi . \sqrt{\frac{E}{f_y}},$$

where:
E = 210 000 MPa is a modulus of elasticity of steel.

When the inequality (2.1) is not matched, the design buckling capacity of the beam could be assumed equal to:

$$(2.4)\quad M_{b,Rd} = k_{fl} . \chi . M_{c,Rd} \leq M_{c,Rd},$$

where:
k_{fl} = 1,1 is coefficient of correction, considering that the method of equivalent compressed flange is aiming to increase the security .
χ - coefficient of buckling of composed "T" – section, out of plain of the web.

In some cases when using this simplified methodology, the inequality (2.1) is matched, but determined according to the (2.4) value of $M_{b,Rd} < M_{y,Ed}$ i.e. , the inequality (2.1) is not satisfied. The Table 1 shows several examples of this type. The question that arises in this case is "Where is the mistake?". In the inequality (2.1) or when determining of $M_{b,Rd}$ with the equality (2.4)? To answer this question author used the general methodology of EN1993-1-1 [2] for determining the overall stability of the beams during bending. The essential here is to determine the critical moment M_{cr} which has been done with analytical and FEA methods.

Table 1. Calculated bearing capacity of $M_{b,Rd}$ of beams, determined according to the simplified methodology

Section	**IPE 200**	**IPE 240**	**IPE 300**	**IPE 360**	**IPN 160**	**IPN 220**	**IPN 280**
distance L_c, m	6	6,3	6,6	7,4	5	5,5	6,2
$M_{y,Ed}$, kN.m	10	19	41,5	68	5,5	16	33
$\bar{\lambda}_f = \frac{k_c . L_c}{i_{f,z} . \lambda_1}$	2,28	1,99	1,67	1,66	2,58	2,15	2
$\bar{\lambda}_{c,0} . \frac{M_{y,Rd}}{M_{y,Ed}}$	2,47	2,16	1,69	1,68	2,77	2,27	2,14
$M_{b,Rd}$, kN.m	8,5	17,9	40,8	67,4	4,2	13,8	30,4

3. Analytical determination of M_{cr}

The removed analytical methodology for determining of the critical moment M_{cr} was mentioned in the preliminary versions of the standard - prEN1993-1-1:2002 [1]. It is valid for sections with 2 axis of symmetry, having one constant section on their length. For instance - hot rolled I-sections with equal flanges. The ends of the beams are restrained against the lateral moving and their torsion is prevented. Generally M_{cr} is determined according to:

$$M_{cr} = C_1 \frac{\pi^2.E.I_z}{(k.L_c)^2}.\left[\sqrt{\left(\frac{k}{k_w}\right)^2.\frac{I_w}{I_z} + \frac{(k.L_c)^2.G.I_t}{\pi^2.E.I_z} + (C_2.z_g)^2} - C_2.z_g\right], \tag{3.1}$$

where:

G is shear modulus of steel;

k – coefficient of the buckling length. Considering the possibility of rotation of ends of the beam in the horizontal plain surface i.e. if there is a possibility to rotate the ends toward the "weak" axis "z-z". Its values can be 0,5; 0,7 and 1,0 (by analogy with the compressed frame);

k_w – coefficient of the buckling length. Considering the possibility for out of plane deformation in the ends. Its values can are 0,5; 0,7 and 1,0; weather it is free or limited;

I_z – moment of inertia of the beam about axis "z-z" ;

I_w – warping constant;

I_t – torsion constant of the beam;

C_1, C_2 and C_3 are coefficients depending on different load and end restraint conditions on the length L_c, between points of lateral restraints.

z_g – distance between the point of apply of transverse loads and shear centre.

The maximum values of the loads which beams can resist without buckling, calculated by relevant methodology, are shown on the Table 2,

In which:

$M_{c,Rd}$ is design resistance of steel section of bending;

$M_{b,Rd,1}$ - design resistance moment of bended element, calculated according to the general methodology. M_{cr} is analytically calculated , according to the formulae (3.1);

$M_{b,Rd,2}$ - design resistance moment of bended element, calculated according to the simplified methodology.

Table 2. Design resistance moment $M_{b,Rd}$ of the beams determined by general and simplified methodologies

Section	**IPE 200**	**IPE 240**	**IPE 300**	**IPE 360**	**IPN 160**	**IPN 220**	**IPN 280**
distance L_c, m	6	6,3	6,6	7,4	5	5,5	6,2
$M_{y,Ed}$, kN.m	10	19	41,5	68	5,5	16	33
$M_{c,Rd}$, kN.m	49,5	82,1	140,6	228,1	30,4	72,5	141,4
$M_{b,Rd,1}$, kN.m	16,71	29,2	48,2	70,8	11,36	28,61	57,4
$M_{b,Rd,2}$, kN.m	8,5	17,9	40,8	67,4	4,2	13,8	30,4

The calculated examples show that when distances L_c between lateral restraints are large, the simplified methodology is too rigorous compared to the general. Table 1 и Table 2 show that if inequity (2.1) is matched, the beams should not lose overall stability.

When distances L_c between lateral restraints of the beams are small, always when the inequity (2.1) is matched, it is matched also (1.1). Examples of those types of beams are shown in Table 3 и Table 4.

Table 3. Design resistance moments $M_{b,Rd}$ of the beams, determined according to the general and simplified methodology. There is a large distances between lateral restraints

Section	**IPE 200**	**IPE 240**	**IPE 300**	**IPE 360**	**IPN 160**	**IPN 220**	**IPN 280**
distance L_c, m	4	4,2	4,3	4,5	3	3,2	3,5
$M_{y,Ed}$, kN.m	16	30	63	110	9,5	28,5	60
$\bar{\lambda}_f = \dfrac{k_c.L_c}{i_{f,z}.\lambda_1}$	1,52	1,33	1,09	1,01	1,55	1,25	1,13
$\bar{\lambda}_{c,0}.\dfrac{M_{y,Rd}}{M_{y,Ed}}$	1,55	1,37	1,12	1,04	1,6	1,27	1,18
$\chi_{f,z}$	0,308	0,378	0,489	0,535	0,3	0,411	0,468
χ_{LT}	0,446	0,458	0,447	0,414	0,514	0,54	0,558
$M_{c,Rd}$, kN.m	49,5	82,1	140,6	228,1	30,4	72,5	141,4
$M_{b,Rd,1}$, kN.m	22	37,6	62,8	94,5	15,6	39,2	78,9
$M_{b,Rd,2}$, kN.m	16,7	34,2	75,6	134,3	10	32,7	72,8

Table 4. Design resistance moments $M_{b,Rd}$ of the beams, determined according to the general and simplified methodology. There is a small distances between lateral restraints

Section	**IPE 200**	**IPE 240**	**IPE 300**	**IPE 360**	**IPN 160**	**IPN 220**	**IPN 280**
distance L_c, m	2	2,1	2,2	2,45	1,65	1,85	2,05
$M_{y,Ed}$, kN.m	32	61	124	203	17	50	100
$\bar{\lambda}_f = \dfrac{k_c.L_c}{i_{f,z}.\lambda_1}$	0,76	0,66	0,56	0,55	0,85	0,72	0,66
$\bar{\lambda}_{c,0}.\dfrac{M_{y,Rd}}{M_{y,Ed}}$	0,77	0,67	0,57	0,56	0,9	0,73	0,71
$\chi_{f,z}$	0,687	0,748	0,81	0,815	0,631	0,711	0,748
χ_{LT}	0,637	0,616	0,63	0,56	0,664	0,67	0,68
$M_{c,Rd}$, kN.m	49,5	82,1	140,6	228,1	30,4	72,5	141,4
$M_{b,Rd,1}$, kN.m	31,5	50,6	88,6	127,7	20,2	48,6	96,2
$M_{b,Rd,2}$, kN.m	37,4	67,5	125,2	204,5	21,1	56,7	116,3

4. Determination of M_{cr} using FEA

For evaluation of results, calculated with the analytical methods, the author did FEA research of the steel beams. For this purpose he used the software SAP 2000 v.14 with its option Buckling analysis. Researched are hot rolled sections **IPE200** and **IPE300**. They are modeled according to the:

- flanges and web of beams are modeled with shell elements with thickness, equal to the mentioned in the standard EU 19-57;
- the loads act on the upper flange, in joints of shell elements, simulating the equal distribution of the load on the beam;
- horizontal restarints of the beam are put in the middle of the upper flange. They prevent only the horizontal movement perpendicular to the beam's axis;
- the used steel is S235, with characteristics according to the standard EN 10025;
- modulus of elasticity $E = 2,1.10^8$ **kPa**;
- coefficient of Poisson $\nu = 0,3$.

Using option Buckling analysis for various beams, are determined critical values of the loads and by them - critical bending moments M_{cr} of the beams. After that, by formulae (1.2) ÷ (1.5) are calculated also the design resistance moments $M_{b,Rd,3}$ of the beams when they going to lateral - torsional buckling. The results are shown in Table. 5.

Table 5. Comparison of the of calculated design resistance moments $M_{b,Rd}$ with beams, determined with analytical and FEA methods.

Section	**IPE 200**			**IPE 300**		
distance L_c, m	6	4	2	6,6	4,4	2,2
$M_{b,Rd,1}$ – general method	16,71	22	31,5	48,2	62	88,6
$M_{b,Rd,2}$ – simplified method	8,5	16,7	37,4	40,8	73,5	125,2
$M_{b,Rd,3}$ – FEA method	17,78	21,87	37,41	42,37	74,87	105,5

5. Conclusion

The calculated examples show that when the distance L_c between the lateral supports of the beam is large, the simplified methodology is too strong comparing to the general one. When the inequity (2.1) is matched it means that the beam would not lose overall stability, even if the inequity (1.1) is not matched.

When the distance L_c between the lateral supports of the beam is small, always when the inequity is matched (2.1), it is matched also the inequity (1.1).

Determined by FEA methods design resistance moment $M_{b,Rd,3}$ of the beam during the lateral-torsional buckling is an envelope of the calculated by another methods values of bearing capacity $M_{b,Rd,1}$ and $M_{b,Rd,2}$.

LITERATURE:

[1] prEN 1993-1-1:2002, Eurocode 3: Design of steel structures – Part 1-1: General rules
[2] EN 1993-1-1:2005, Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings.