FORCES IN OPEN ROAD STEEL STRUCTURE OF RAILROAD BRIDGE, DETERMINED ANALYTICALLY AND NUMERICALLY

Lyubomir Zdravkov[1]

University of Architecture, Civil Engineering and Geodesy – Sofia

Abstract: *Open road structure of steel railway bridges is constructed of longitudinal (LB) and transverse (TB) beams. The longitudinal beams pass over or through the transverse beams and transmit their vertical loads through common joints. Since nodes are rigid, i.e. they provide mutual rotation of items, LB and TB form plain rigid structure that is statically indeterminate system. Internal forces in elements of plain rigid structure caused by vertical moving loads depends on the location of loads on LB, and the ratio of the bending stiffness of LB and TB.*
Forces in steel members of plain structure could be determined analytically, using simplified methods, or precisely, using FEA. Main question is how much is difference between two methods?

Key words: *railroad bridge, opened road structure, forces, FEA*

Open type road structure of steel railroad bridges is built by longitudinal (*LB*) and transverse (*TB*) beams. When bridge has only one railway track, longitudinal beams are two and the distance between them usually is ***b*** = 1,70 ÷ 1,80 m [2]. The distance ***a*** between transverse beams which supports the longitudinal beams depends on the type of the main girders. When main girders are plated girders the distance is ***a*** = 4,0 ÷ 5,5 m [2].

The longitudinal beams pass above or through transverse beams and through their common joints transfer to them loads. As the common joints are stiffened i.e. they assure the same rotation of the elements, the longitudinal and transverse beams form open rigid plain structure which is a statically undetermined system. Internal forces in elements in the rigid road structure, caused by vertical moving loads, depend on location of the loads on *LB*, but depends also on ratio of the bending stiffness of longitudinal and transverse beams.

1. Determining of internal forces in the longitudinal and transverse beams

Based on the multiple statistical indetermination of the open rigid plain structure of railroad, the approaches for determination of internal forces and correct sections of the *LB* and *TB* are as follow:

a) creating numerical 3D model of the open rigid plain structure, using some of existing software for structural analysis. Disadvantage of this approach is that multiple calculations and approaches are needed for calculating internal forces in its elements and their measurements. It is due because on fact that correlation of bending stiffness of longitudinal and transverse beams is very important for internal forces in them, but we do not know in advance what steel sections to use in the model. ;

b) determining in advance analytically internal forces in *LB* and *TB,* using the methods described in specialized literature [1]. Disadvantage of this method is that it is not clear whether the used here method is correct and gives the correct results.

The purpose of current research is to check whether this old analytical methodology is reliable.

1.1 Internal forces in longitudinal beams, analytical methodology

As a result of vertical deformation of the transverse beams for vertical moving loads, longitudinal beams have statical scheme multi-span beams on elastic (spring) supports, see Fig. 1.

[1] Lyubomir Zdravkov, PhD, associate professor, civil engineer, UACEG, Sofia 1046, №1 „Hristo Smirnensky" str., floor 7, office 733, e-mail: zdravkov_fce@uacg.bg

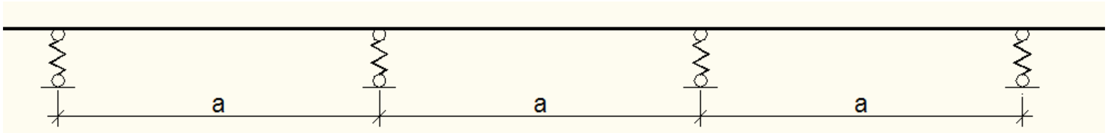

Fig. 1. Statical scheme of longitudinal beam for vertical moving loads.

Considering the difficulties to calculate internal forces in the longitudinal beams when using its real statical scheme, at the level of first approximation according to the prescriptions in [1] , we can use the analogy with simply-supported beam, see Fig. 3. Distance between supports ***a*** is equal to the distance between two transverse beams.

The internal forces in the substituting simply-supported beam could be calculated according to the formulas:

- from the dead (permanent) loads on the longitudinal beam Σg_i :

(1.1) $$M_{G,max} = \frac{1}{8} \cdot \sum g_i \cdot a^2$$

(1.2) $$Q_{G,max} = \frac{1}{2} \cdot \sum g_i \cdot a$$

- moving vertical loads on the longitudinal beam:

(1.3) $$M_{Q,max} = k_e \cdot Q_{vk} \cdot \sum \eta_{M,i} \cdot \gamma_Q$$

(1.4) $$Q_{Q,max} = k_e \cdot Q_{vk} \cdot \sum \eta_{Q,i} \cdot \gamma_Q ,$$

where:

k_e is a coefficient reporting the eccentricity of the vertical loads, see Fig. 2;

Q_{vk} – characteristic value of vertical loads from rail vehicles;

$\eta_{M,i}$ – ordinate of the line of influence of bending moments, see Fig. 3 – б ;

$\eta_{Q,i}$ – ordinate of the line of influence of shear forces, see Fig. 3 – в ;

γ_Q – coefficient of loading of vertical moving loads.

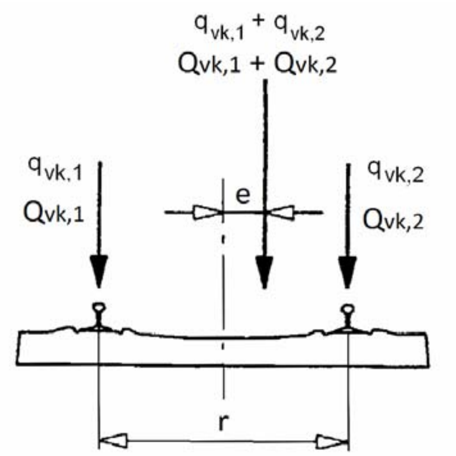

Fig. 2. Eccentricity of vertical loads

$$\alpha.Q_{vk} = Q_{vk,1} + Q_{vk,2}$$

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$$\frac{Q_{vk,2}}{Q_{vk,1}} \leq 1{,}25 \qquad \frac{q_{vk,2}}{q_{vk,1}} \leq 1{,}25$$

$$e \leq \frac{r}{18} \approx 8{,}3cm ,$$

where:

r = 150 cm is distance between axes of rails

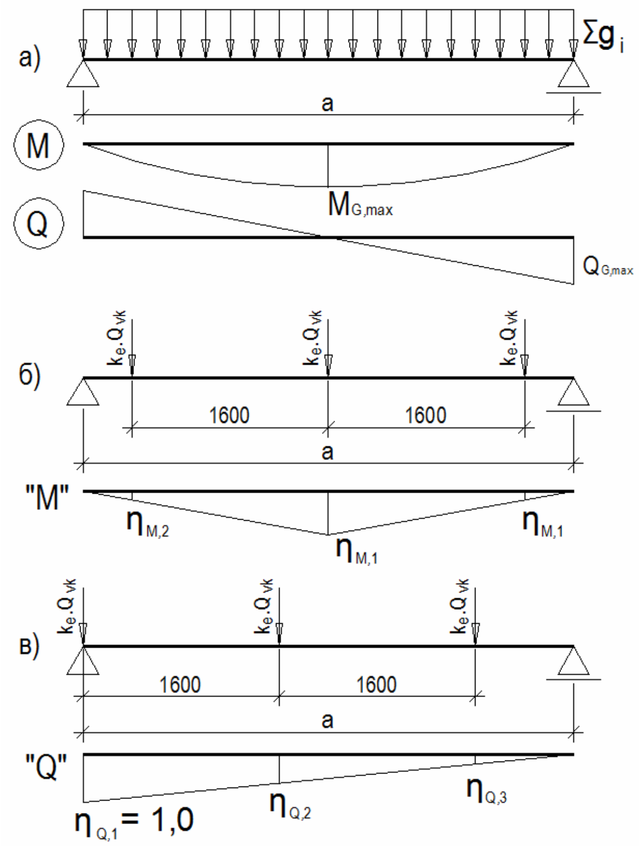

Fig. 3. Substituting simply-supported beam

а) location of the dead (permanent) loads | б) location of the moving loads for maximum moment $\boldsymbol{M}_{\mathrm{Q}}$ | в) location of the moving loads for maximum shear forces $\boldsymbol{Q}_{\mathrm{Q}}$

Based on the forces, calculated with substituting simply-supported beam, the internal forces in the multi-span longitudinal beam on elastic (spring) supports would be:

(1.5) $M_m = 0{,}8.(M_{\mathrm{G,max}} + \Phi_{\mathrm{i}}.M_{\mathrm{Q,max}})$ - maximum bending moment in the field of *LB*;

(1.6) $M_o = 0{,}75.(M_{\mathrm{G,max}} + \Phi_{\mathrm{i}}.M_{\mathrm{Q,max}})$ - bending moment in *LB* over the elastic support;

(1.7) $V_{\mathrm{z,Ed}} = Q_{\mathrm{G,max}} + \Phi_{\mathrm{i}}.Q_{\mathrm{Q,max}}$ - maximum vertical shear force,

where:

Φ_{i} e dynamic coefficient for loading by rail vehicles, depending on railroad maintaining.

For a line with standard maintaining i.e. $\Phi_{\mathrm{i}} = \Phi_3$, according to the European standard EN 1993-2, this coefficient is determined according to the formulae:

(1.8) $$\Phi_3 = \frac{2{,}16}{\sqrt{L_\Phi} - 0{,}2} + 0{,}73, \text{ where} 1{,}00 \le \Phi_3 \le 2{,}0 ,$$

in which:

L_Φ is a "determining" length, correlated with Φ_3, in [m].

1.2 Internal forces in the transverse beams by vertical loads, analytical methodology

Transverse beams have statical scheme simply-supported beam with span ***B***, equal to the distance between main girders, see Fig.5.

For determining of forces in the transverse beams caused by moving loads, multi-span longitudinal beam on elastic supports is considered as two jointed simply-supported beams [1]. The line of influence for support's reaction and position of loads are shown on the Fig. 4.

The constructed on this manner line of influence determines the characteristic value of reaction R_v^n on supports:

(1.9) $$R_v^n = k_e.Q_{vk}.\sum \eta_{R,i} + k_e.q_{vk}.\sum A_{R,i} ,$$

where:

$\eta_{R,i}$ is an ordinate of line of influence for reaction of support, see Fig. 4 ;

q_{vk} - characteristic value of evenly distributed vertical load caused by railroad vehicle, estimated for one rail;

$A_{R,i}$ – the field of line of influence, "closed" by load q_{vk}

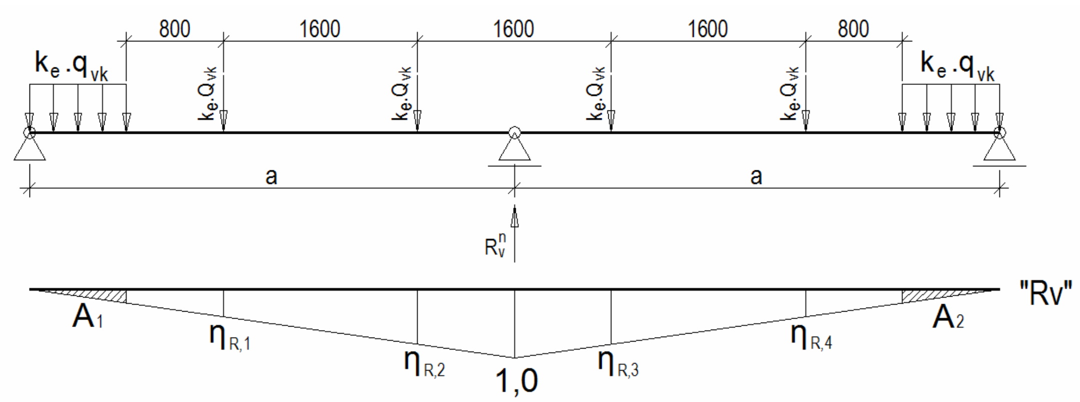

Fig. 4. Line of influence for support's reaction R_v^n

Design value of reaction of support by vertical loads which will load transverse beam is:

(1.10) $$R_v = R_v^n.\Phi_i.\gamma_Q ,$$

The distribution of the vertical load on the length of transverse beam and internal forces caused by them are shown on Fig. 5.

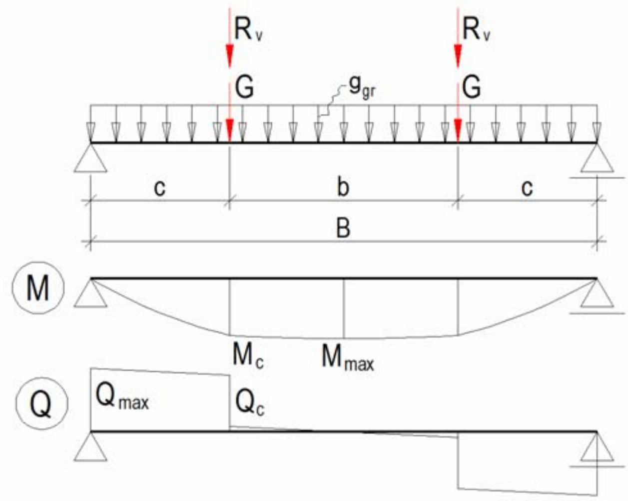

Fig. 5. Vertical loads on transverse beam and internal forces in it

1.3 Internal forces in longitudinal and transverse beams caused by vertical loads, numerical calculating model

In order to have reliable results using numerical model we have to use the correct sections of the longitudinal and transverse beams. Two approaches can be used for their preliminary determining:

- Determining the forces in the elements of the open rigid plain structure, using the above mentioned analytical method;

- Using database of designed before similar bridges which can provide the sections of the longitudinal and transverse beams.

After the determination of sections of beams, their bending stiffness are known and we can create numerical calculating model of the open road rigid structure, using appropriate software.

Dead (permanent) loads caused by self-weight of rails and crosspieces are inserted as evenly distributed load acting on the longitudinal beams. Beams weight is automatically inserted by software. The loads caused by railroad vehicles are inserted on the longitudinal beams with their eccentricity. Their distribution on the length of the bridge must be such as the distribution we need to calculate maximum values of the internal forces and deformations in the researched elements.

2. Comparison of calculated values of internal forces

The results are obtained with the help of several groups of students, specialty "Structural Engineering" who made researches for their course project by "Steel Bridges". Having in mind the staff which conducted the study, obtained results could not be considered as completely reliable but can show the general trend.

Part of the obtained results is shown in the Table 1.

The difference in the values of internal forces obtained by analytical and numerical methods are:

– for bending moments in field $M_{поле}$ in the longitudinal beams, with two exceptions are smaller than 7% . Average values of all differences – 7,76% ;

- for bending moments in the longitudinal beams in the joints with transverse beams (above the supports) $M_{опора}$, with one exception are bigger than those calculated numerically. The difference in their values is not more than 45%. Average values of all differences – 21,87%.

- for shear forces Q in the longitudinal beams, with two exceptions is smaller than 25 % . Average values of all differences – 15,14 %.

- for bending moments in the transverse beams, with one exception is smaller than 9 % . Average values of all differences – 6,15%.

- for shear forces Q transverse beams, with one exception is smaller than 17 % . Average values of all differences - 10,04 %.

Table 1. Values of bending moments and shear forces in the longitudinal and transverse beams

Element	**Lengthwise beam**			**Transverse beam**	
Solution	$M_{поле}$	$M_{опора}$	Q	M	Q
FEA	573,4	424	683,8	2266	1082
Analytical	567,6	532,2	734,7	2131	1018
Difference, %	1,02	25,5	7,44	6,32	6,23
FEA	568,2	462,1	689,8	2008	1068
Analytical	551,5	517,0	651,3	2186	1123
Difference,%	3,04	11,9	5,91	8,88	5,15
FEA	628,5	455,3	605,8	2366	1186
Analytical	628,2	588,9	577,2	2242	1124
Difference,%	0,06	29,34	4,94	5,54	5,50
FEA	520,6	423,8	577,5	2033	1023
Analytical	553,2	518,6	719,8	2018	1094
Difference,%	6,26	22,36	24,64	0,73	6,90
FEA	370	250	497,5	1091	644
Analytical	380	356	669,1	1275	752
Difference, %	2,70	42,40	34,49	16,87	16,7

Element	**Lengthwise beam**			**Transverse beam**	
Solution	$M_{поле}$	$M_{опора}$	Q	M	Q
FEA	193	173	437	1451	766
Analytical	191	179	413	1341	709
Difference,%	1,05	3,47	5,81	8,20	8,04
FEA	673,7	568,4	606,7	1099	1093
Analytical	490,3	459,7	714,5	1105	1005
Difference,%	37,4	23,6	17,76	0,55	8,72
FEA	349,4	266,4	593,8	1532	959,4
Analytical	422,8	273	679,1	1565	980,6
Difference,%	20,98	2,45	14,38	2,15	2,21
FEA	534,6	344,4	554,7	1900	1057
Analytical	526,3	493,4	712,4	1746	1454
Difference,%	1,58	43,27	28,4	8,76	37,51
FEA	565,2	447,5	693,3	1840	1085
Analytical	545,9	511,7	644,	1904	1123
Difference,%	3,55	14,4	7,65	3,47	3,48

3. Conclusions

Comparing the values of internal forces in the elements of the open rigid plain structure we can mention the following trends:

- Bending moments in the longitudinal beams in common joint with the transverse beams (over the supports) , obtained by analytical methodology are bigger than obtained using FEA;
- The differences in the values of bending moments and shear forces are not too big;
- Preliminary determination of the sections in the beams using the mentioned in [1] analytical methodology which later will be used in FEA is an acceptable possibility;
- It is not advisable to use only analytical methodology for determining forces in the elements in the open rigid plain structure. Numerical method is absolutely necessary.

Literature

[1]. Димитров Д., П. Стайков, Ст. Стойнов, и др., Ръководство за упражнения по стоманени мостове, София, 1976.
[2]. Дулевски Енчо, Стоманени мостове, Второ издание, София, 2006.