USE OF LBA-MNA METHODOLOGY FOR DETERMINATION OF BEARING CAPACITY OF COMPRESSED SHELLS

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***Abstract**: Using GNIA or GMNIA in new, still unmade shells is related with a large scope, heavy and continuous calculations. Main difficulty is modeling of imperfections with correct values. Ordinary designers who are usually hard pressed by time and are not familiar with computing software, enabling full GNIA or GMNIA analysis of shells, require simplified procedures in which imperfections are not included. Such a procedure, called LBA-MNA, is presented here. It allows analysis of real shells, modelled as perfect structures.*

***Key words**: shell, compression, buckling, imperfections, LBA, MNA*

1. Introduction

During the analysis of imperfect shells considering geometric nonlinearity when material is elastic (**GNIA**) or of imperfect shells considering the geometrical and material nonlinearity (**GMNIA**), in the calculated model should be introduced a set of imperfections. Unfotunately the last one may have a lot of forms and amplitudes. Determining their most unfavorable form and dimensions is a real challenge [3]. It is important that imperfections will be realistic, having in mind the way of producing of the steel shells. For example, current formulas for manual calculation are obtained empirically, after a lot of laboratory tests through which is determined the lowest bearing capacity of the shell, before buckling. Due to the fact that the examined samples are produced in the laboratories, their quality, respectively - their imperfections, are quite different from full-size, real shells [4].

In general case, imperfections decrease the bearing capacity of the shell. Sometime, when the imperfection is very high, the last one can increase stability of shell. As an example can be shown a deeper imperfections which can cause the loss of stability on the whole length of the shell [2].

When the shells are made out from steel, the process to find out which are their the most unfavourable imperfections is further more complicated, because of many forms of warp which influence to each other, which leads to different sensibility to the geometrical imperfections. For example a lot of cylindrical shells bear the combined pressure from axial and radial loads. The imperfections which provoke the lowest bearing capacity during the meridional pressure are completely different from this one for radial pressure. The first one are local, while the other should be despatched on the biggest length. Adding that real shells bear the unevenly applied or concentrated loads, it become very difficult to determine which is more unfavourable form of imperfections [4].

Obviously GNIA or GMNIA are more convenient for analysis of shells which are already built, because their imperfections are unambiguously determined in the process of their construction. The use of GNIA or GMNIA for new, already non-constructed shells require a lot of work, heavy and continuous calculations. The designers who must meet the short deadlines and who do not know well the calculating programs allowing true GNIA or GMNIA of shell, need more simple procedure. Such a procedure, called LBA-MNA which meets the requirements of standard

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БДС EN 1993-1-6 [1] is described from *Schmidt* and *Rotter* [5] in their article from 2008. Briefly the procedure can be described in the following steps:

- research model of the shell is created using a program for analysis of structures. It is perfect, without geometrical flaws shell. Material is elastic;
- to the model is applied load, which we expect to cause compressive stresses on the shell, respectively, leads to loss of stability;
- option "Buckling Analysis" of a program is activated and do a linear-elastic bifurcation analysis (**LBA**). It establishes what is the reserve of bearing capacity of the shell before buckling;
- inserted load, multiplied by reserve of bearing capacity will give the value of the critical force in the shell, in which it will loses stability in elastic stage. This critical force is marked as R_{cr};
- load - bearing capacity of the shell in plastic stage R_{pl} should be determined. It is calculated through analysis with a non-linear behavior of the material (**MNA**). The maximum value of the stresses is equal to the yield strength of steel. In shells with longitudinal axis of symmetry loaded with a uniformly distributed load, obtaining of R_{pl} is simple and can be done by the formulas of Annex A or B of standard БДС EN 1993-1-6 [1].
- relative slenderness of the shell should be calculated, by formulae:

$$\bar{\lambda} = \sqrt{\frac{R_{pl}}{R_{cr}}} \quad (1.1)$$

- after that follow the instructions of БДС EN 1993-1-6 for determining the value of the coefficient of loss of stability χ.

2. Applicability of LBA - MNA methodology when using SAP 2000

To illustrate the above mentioned concepts, as well as for assessing the viability of the method when using program SAP 2000 [6], is solved example with a cylindrical steel shell. Its geometry is shown in Fig. 1. The shell is made of steel S235 and has following boundary conditions:

- lower edge - hinges in supports, where the possibility of meridional and radial displacement is restrained;
- upper edge - stiffening ring is placed. It allows rotating and displacement in meridional direction. There is no possibility for radial movement.

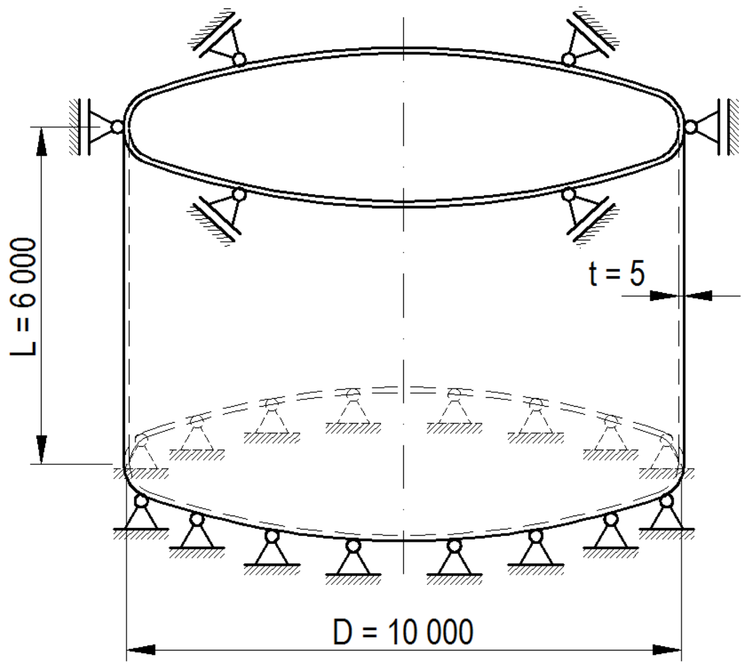

Fig. 1. Cylindrical steel shell. Dimensions and supporting conditions

Cylindrical shell is loaded with axial (meridional) compression forces.

The purpose of research is to be determined the limits of normal meridional stresses where the cylinder will loses stability.

2.1 Analytical (manual) solution, according to БДС EN 1993-1-6

Length of the segment of cylindrical shell could be characterized with dimensionless parameter ω:

(2.1) $$\omega = \frac{L}{R}.\sqrt{\frac{R}{t}} = \frac{L}{\sqrt{R.t}} = \frac{6000}{\sqrt{5000.5}} = 37{,}947\,,$$

where:

L = 6000 mm is a length of cylindrical shell, see Fig. 1;

R = 5000 mm - radius of cylindrical shell;

t = 5 mm - thickness of shell.

(2.2) $$1{,}7 < \omega = 37{,}947 < 0{,}5.\frac{R}{t} = 500 \rightarrow \text{medium - length cylinder.}$$

Elastic critical meridional buckling stress $\sigma_{x,Rcr}$ could be calculated by formulae:

(2.3) $$\sigma_{x,Rcr} = 0{,}605.E.C_x.\frac{t}{R} = 0{,}605.21000.1.\frac{0{,}5}{500} = 12{,}705\ \mathrm{kN/cm^2}\,,$$

where:

E = 21 000 kN/cm^2 is a modulus of elasticity of steel;

C_x - coefficient, depending on length ω . For medium - length cylinders, C_x = 1.

Characteristic values of critical meridional buckling stress $\sigma_{x,Rk}$ could be calculated by multiplying of yield strength of steel f_y with a coefficient of buckling χ_x:

(2.4) $$\sigma_{x,Rk} = \chi_x.f_y = 0{,}055.23{,}5 = 1{,}292\ \mathrm{kN/cm^2}$$

Coefficient of buckling χ_x should be determined as a function of the relative slenderness of the shell $\bar{\lambda}_x$ from:

(2.5) $$\text{when } \bar{\lambda}_{xp} \le \bar{\lambda}_x \rightarrow \chi_x = \frac{\alpha_x}{\bar{\lambda}_x^2} = \frac{0{,}1017}{1{,}36^2} = 0{,}055\,,$$

where:

$\bar{\lambda}_x$ is a elastic relative slenderness of shell;

$\bar{\lambda}_{xp}$ - plastic limit relative slenderness of shell;

α_x - meridional elastic imperfection reduction factor.

(2.6) $$\bar{\lambda}_x = \sqrt{\frac{f_y}{\sigma_{x,Rcr}}} = \sqrt{\frac{23,5}{12,705}} = 1,36$$

(2.7) $$\bar{\lambda}_{xp} = \sqrt{\frac{\alpha_x}{1-\beta}} = \sqrt{\frac{0,1017}{1-0,60}} = 0,504\,,$$

in which:
$\beta = 0,60$ - plastic range factor.

(2.8) $$\alpha_x = \frac{0,62}{1+1,91.\left(\frac{\Delta w_k}{t}\right)^{1,44}} = \frac{0,62}{1+1,91.\left(\frac{0,988}{0,5}\right)^{1,44}} = 0,1017\,,$$

where:
Δw_k is characteristic imperfection amplitude. Should be calculated as follow:

(2.9) $$\Delta w_k = \frac{1}{Q}.\sqrt{R.t} = \frac{1}{16}.\sqrt{500.0,5} = 0,988\,,$$

where:
Q is meridional compression fabrication quality parameter. Q = 16 for normal quality.

Design buckling stress $\sigma_{x,Rd}$ should obtained from:

(2.10) $$\sigma_{x,Rd} = \frac{\sigma_{x,Rk}}{\gamma_{M1}} = \frac{1,292}{1,1} = 1,175 \text{ kN/cm}^2\,,$$

where:
γ_{M1} = 1,1 is partial factor for resistance to buckling.

2.2 Numerical solution with using of FEA

Using software SAP 2000 [6] is created spatial computing model of perfect cylindrical shell. The dimensions and thicknesses of the elements, as well as the boundary conditions are in accordance with the scheme, shown on Fig. 1. The cylinder is divided (meshed) into a plurality of shell elements, such as variations on the theme are shown in Table 1. At each node in the upper ring are applied axial compressive forces, each with a value of 1,0 kN. Their sum gives the resultant axial compressive force on the cylinder. The option “Buckling Analysis” is activated. This option may calculate a reserve of carrying capacity ***k*** of the cylindrical shell before buckling, partly or entirely. The total compressive force, multiplied by reserve of bearing capacity ***k*** gives the value of critical force R_{cr}, in which the ideal shell will lose stability in elastic stage.

From Table 1 it is clear that different meshing of shell get different values of critical force R_{cr}. In general, the smaller components lead to more accurate results.

Table 1. Results of numerical solution

Mesh of cylindrical shell		Compression force, kN	Coefficient of overloading ***k***	R_{cr}, kN
radially	meridional			
32	6	32	2420,04	77 441,3
60	10	60	579,18	34 750
180	30	180	185,37	33 366
360	36	360	56,029	20 170,4

Plastic resistance of shell R_{pl} on meridional (axial) compression should be obtained from:

(2.11) $R_{pl} = A_{нс} . f_y = 1570{,}8 . 23{,}5 = 36913{,}8 \text{ kN}$,

where:
$A_{нс}$ = 1570,8 cm^2 is area of section of cylindrical shell;
f_y = 23,5 kN/cm^2 - characteristic value of yield strength of used steel .

Relative slenderness of shell $\bar{\lambda}_x$ cold be calculated by formulae :

(2.12) $\bar{\lambda}_x = \sqrt{\frac{R_{pl}}{R_{cr}}} = \sqrt{\frac{36913{,}8}{20170{,}4}} = 1{,}353$

(2.13) $\bar{\lambda}_x = 1{,}353 > \bar{\lambda}_{xp} = 0{,}504 \rightarrow$

(3.14) $\rightarrow \chi_x = \frac{\alpha_x}{\bar{\lambda}_x^2} = \frac{0{,}1017}{1{,}353^2} = 0{,}0556$

(2.15) $\sigma_{x,Rk} = \chi_x . f_y = 0{,}0556 . 23{,}5 = 1{,}306 \text{ kN/cm}^2$

(2.16) $\sigma_{x,Rd} = \frac{\sigma_{x,Rk}}{\gamma_{M1}} = \frac{1{,}306}{1{,}1} = 1{,}187 \text{ kN/cm}^2$.

Difference with a analytical solution is 1,04 %.

3. Conclusions

The use of the simplified procedure LBA-MNA, proposed by *Schmidt* and *Rotter* [5] could successfully cancel the need to apply GNIA or GMNIA by ordinary designers in the analysis of shells. In LBA-MNA are not required expensive specialized programs, nor excessive knowledge and skills in simulation of studied objects. Not least, it should be noted that thus saving valuable time of designers.

Made from several numerical models that could not be shown due to the limited volume of the article, it is clear that the differences between the analytical and numerical solution are not too high. For example:

- centrically compressed cylinder in meridional direction - 1,04 %
- centrically compressed cylinder in radial direction - 28,56 %
- cylinder, subjected on shear - 11,47 % difference.

LITERATURE

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