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Self - supporting Dome Roof on Tank with V = 70 000 m^3 capacity. New approaches to design

Lyubomir A. Zdravkov

PhD, Associate Professor, Civil Engineer
University of Architecture, Civil Engineering and Geodesy, Faculty of Civil Engineering, №1 "Hristo Smirnensky" str., floor 7, Sofia 1046, Bulgaria
e-mail: zdravkov_fce@uacg.bg

A French company, in performance with program for expansion of its production, assigns design of tank for molasses with V = 70 000 m^3 capacity and self-supporting dome roof. The tank will be erected in company's storage area, near to Dobrovice, Czech Republic, next to other two, the same tanks, with V = 70 000 m^3 capacity. One of the requirements of Investor is to reduce weight of steel structure, especially weight of the roof. Main reason for that are very heavy sections, according to the point of view of Investor. That requirement forces design to be as light as possible, but on safe side. Of special interest is calculation of the elements of roof structure and possible new approaches to demonstrate their bearing capacity.

Keywords: *design, Laplace, roof cover plates, roof's structure, self-supporting dome roof*

1. Basic geometrical data of the tank

Diameter of shell - D = 64,0 m;
Height of shell - H = 22,020 m;
Height of roof - f = 5,49 m;
Radius of bending of dome roof - R_r =1,5.D=1,5.64=96,0m;
Number of radial girders in the roof - n_r = 100 pcs.;
radial girders in central ring of the roof - 20 pcs.
According to real project, radial girders are with section IPE 220.
Circular elements - none, see Fig. 1;
Thickness of roof cover plates - t_r = 5 mm;
Top angle on the shell - angle section L200x12;
Roof structure and cover plates are made by steel S355.

Fig. 1. *Aboveground steel tank with V = 70 000 m^3 capacity for molasses*

2. Loads on dome roof

- overpressure - p_o = 5 mbar;
- negative pressure (vacuum) - p_v = 2,5 mbar;
- snow load on the ground of area of tank's exploitation (Dobrovice) - s_k = 0,75 kN/m^2;

Snow load s on the roof of tank could be calculated by formulae, see standard EN 1991-1-3:2006:

$$s = \mu_i . C_e . C_t . s_k = 0{,}8.1.1.0{,}75 = 0{,}6 \text{ kN/m}^2 \tag{1}$$

where:
μ_i is the snow load shape coefficient;
C_e – the exposure coefficient;
C_t – the thermal coefficient;
s_k – the characteristic value of snow load on the ground for given location.

- wind - basic wind speed $v_{b,0}$ = 36,12 m/s;

The peak velocity pressure $\boldsymbol{q_p(z)}$ at height z, which includes mean and short - term velocity fluctuations, could be calculated using formula as follow, see standard EN 1991-1-4:2005:

$$q_p(z) = \left[1 + 7.I_v(z)\right].\frac{1}{2}.\rho.v_m^2(z) \text{ , N/m}^2 \tag{2}$$

where:
ρ = 1,25 kg/m^3 is the air density;
v_m(z) - mean wind velocity at height **z** above terrain level;
I_v(z) - the turbulence intensity at height **z**.

Coefficient for external pressure $\boldsymbol{c_{pe}}$ should be defined to determine wind pressure acting on the external surfaces. The coefficient $\boldsymbol{c_{pe}}$ should be accounted according to Fig. 7.12 of standard EN 1991-1-4:2005, in respect of geometrical values of the tank h, d и f:

$$\left.\begin{aligned} \frac{h}{d} &= \frac{22{,}02}{64} = 0{,}344 \\ \frac{f}{d} &= \frac{5{,}49}{64} = 0{,}0858 \end{aligned}\right\} \Rightarrow \left| \begin{aligned} c_{pe,A} &= 1{,}35 \\ c_{pe,B} &= 0{,}6 \\ c_{pe,C} &= 0{,}4 \end{aligned}\right. \tag{3}$$

Wind pressure acting on the external surfaces w_e, should be obtained from expression:

$$w_e = q_p(z_e).c_{pe} \tag{4}$$

$$\text{for p.A} \rightarrow w_{e,A} = q_p(z_{e,A}).c_{pe,A} = 2346{,}95.1{,}35 = 3168{,}4 \text{ N/m}^2$$

$$\text{for p.B} \rightarrow w_{e,B} = q_p(z_{e,B}).c_{pe,B} = 2472{,}95.0{,}6 = 1483{,}8 \text{ N/m}^2$$

$$\text{for p.C} \rightarrow w_{e,C} = q_p(z_{e,C}).c_{pe,C} = 2346{,}95.0{,}4 = 938{,}8 \text{ N/m}^2$$

Mean characteristic value of wind pressure $w_{e,m}$ by whole roof could calculated using formula:

$$w_{e,m} = 0{,}30.w_{e,A} + 0{,}5.w_{e,B} + 0{,}20.w_{e,C} = \\ = 0{,}30.3168{,}4 + 0{,}5.1483{,}8 + 0{,}2.938{,}8 = 1880{,}2 \text{ N/m}^2 \tag{5}$$

3. Loads combinations

Load combinations could be defined on two main categories, as follow:

a) combination of loads acting from top to bottom

$$q_1 = \gamma_{Fg,sup}.g_n + \gamma_Q.s + \gamma_Q.\psi_0.p_v = \\ = 0{,}6.1{,}35 + 1{,}5.0{,}6 + 1{,}5.0{,}6.0{,}25 = 1{,}935 \text{ kN/m}^2 \downarrow \tag{6}$$

where:

$\gamma_{fg,sup}$ = 1,35 is self-weight overloading coefficient, according to EN 1990;

γ_Q = 1,5 - overloading coefficient for temporary loads;

ψ_0 - coefficient for simultaneously working two or more temporary loads, according to EN 1990.

b) combination of loads acting from bottom to top

$$q_2 = \gamma_Q.w_{e,i} + \gamma_Q.\psi_0.p_o - \gamma_{Fg,inf}.g_n \uparrow \tag{7}$$

where coefficients of overloading are:

$\gamma_{fg,inf}$ = 1,0 – self-weight overloading, when acts favourably, see standard EN 1990.

$$\text{for p.A} \rightarrow q_{2,A} = \gamma_{Fw}.w_{e,A} + \gamma_Q.\psi_0.p_o - \gamma_{Fg,inf}.g_n = \\ = 1{,}5.3{,}168 + 1{,}5.0{,}6.0{,}5 - 1.0{,}6 = 4{,}602 \text{kN/m}^2 \uparrow$$

$$\text{mean} \rightarrow q_{2,m} = \gamma_Q.w_{e,m} + \gamma_Q.\psi_0.p_o - \gamma_{Fg,inf}.g_n = \\ = 1{,}5.1{,}88 + 1{,}5.0{,}6.0{,}5 - 1.0{,}6 = 2{,}67 \text{kN/m}^2 \uparrow$$

4. Traditional approach for design of roof's elements

a) roof cover plates

Accepted static scheme of roof plates is a multi-span girder on many supports, as is shown on Fig. 2. Distance between supports is measured where triangle field, transmitting loads to shell, ends.

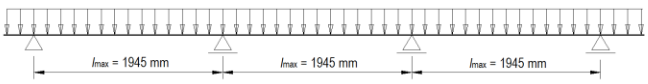

Fig. 2. *Scheme of roof's cover plates*

As a result of research, accepted thickness of roof cover plates is t_{rp} = 5 mm of steel S355, as it is a constructive minimum. So, the thickness by project is the same.

b) roof structure

As the radial girders which are the parts of the dome are eccentric pressured, most important should be verification for general stability loss. Used method in the research is so called "General Method" which has been a little bit modified. Method has been described in standard EN 1993-1-1. A spatial design model of the dome has been created with the software SAP 2000. Option Buckling Analysis is active. This option gives chance to calculate the value of bearing capacity of the construction before it losses stability, partially or entirely, see Fig.3. Used radial girders in spatial roof's model have section IPE 330 from steel S355.

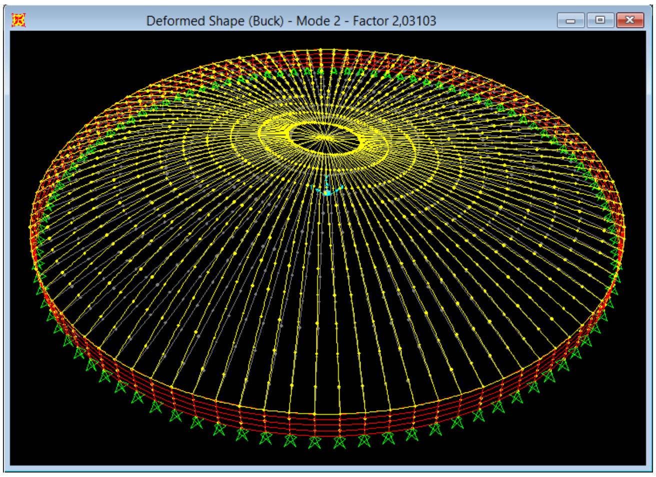

Fig. 3. *Deformed shape of dome roof, that looses stability, due to loading of self-weight g_n, snow s and vacuum p_v*

According to done research, accepted radial girders on the roof with section IPE 330 will not loss stability. Obviously, the calculated section IPE 330 is bigger several times comparing to the real used section IPE 220. Something more – the spherical roof of the tank V = 70 000 m^3 in Dobrovice is executed and put into exploitation several years ago and is still in a good condition. The possible reasons for this difference in the results are as follow:

- the temporary wind, snow and vacuum loads did not reach for now their critical values, which would destroy the spherical roof;

- new, more accurate approach in determining the bearing capacity of the roof's elements.

Second option seems more reasonable. Anyhow, design of such big tanks with V = 70 000 m^3 capacity is not given to the inexperienced companies.

5. New approach for design of roof's elements

The results of done analysis gives to me reason to conclude that classical approaches for design of spatial domes are too conservative. Because of it I begun to look for another alternative approaches for analyses and design of such type of constructions.

a) design of roof's cover plates as a spatial shell

For the roof's cover plates which should be seen as a thin shell with constant thickness, the equation of Laplace could be used:

$$\frac{\sigma_m}{R_m} + \frac{\sigma_r}{R_r} = \frac{p}{t} \qquad (8)$$

where:
σ_m is a normal stress in meridional direction, see Fig. 4;
σ_r – normal tension in the radial (annular) direction ;
R_m – radius of bending in meridional direction;
R_r - radius of bending in radial (annular) direction;
p – the value of the pressure on the shell which can be a function only on the coordinate z;
t - thickness of the shell.

Limitation of the equation (8) is that it can be used in thin shells only, which can be researched according to membrane theory.

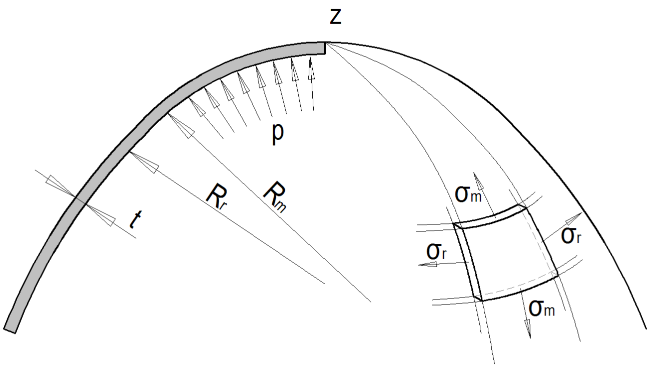

Fig. 4**. Shell with axis of symmetry (rotation) **z** and constant thickness **t

In the spherical shells in which radius of bending in all directions is one, i.e. $R_m = R_r = R$, normal tensions in all direction would be equal. They can be determined according to the formula:

$$\sigma_r = \sigma_m = \frac{p}{2.t}.R \qquad (9)$$

Using it we can calculate minimum of the necessary thickness of the roof's cover plates:

$$t \geq \frac{q_{2,m}}{2.\left(\frac{f_y}{\gamma_{M0}}\right)}.R_r = \frac{2,67}{2.\left(\frac{355000}{1,05}\right)}.96 = 0,0004m = 0,4mm \qquad (10)$$

where:
f_y = 355 MPa - yield strength of steel S355;
γ_{M0} = 1,05 - coefficient of safety by material, according to EN 1993-1-1.

It could be seen that difference in necessary thickness of roof's cover plates, when it is considered as a stiff plate working on pure bending , and as a smooth shell working on pure tension, is much bigger. Several times bigger. Even if we are measuring the roof's cover plates with load combination $q_{2,A}$ for point A , it should be more economical:

$$t \geq \frac{q_{2,A}}{2.\left(\frac{f_y}{\gamma_{M0}}\right)}.R_r = \frac{4,602}{2.\left(\frac{355000}{1,05}\right)}.96 = 0,00065m = 0,65mm \qquad (11)$$

It should be noted that when considering work of the roof's cover plates as a 3-D shell, loaded by combination $q_{2,i}\uparrow$, radial girders are not necessary, i.e. their stress check is satisfied automatically.

Obviously such an approach for considering the work of the roof's cover plates as a 3-D shell is possible when the details for its joints are appropriate for this purpose. For instance, the roof sheets should be joined between themselves through butt welds with full penetration.

Determined by the formula (11) minimum thickness of the roof's cover plates $\boldsymbol{t_{rp}}$ does not correspond with the cases with point loads, caused by people or equipment on the roof. In this case the roof sheets should be considered and designed as plates working on the bending.

b) design of roof structure for loss of stability
- methodology of Evolution Group for EN 1993-1-6

Proposed on by Evolution Group for EN 1993-1-6 methodology should be applied to smooth spherical shells. For this purpose stiffened shell of the spherical roof shall be transformed to equivalent smooth shell which has the same bending stiffness. On the base of equilibrium between the moments of inertia of the roof's cover plates and radial girder we calculate:

$$\frac{1}{12}.a_n.t_{ek}^3 = I_0 \qquad (12)$$

And after clear transformation :

$$t_{ek} = \sqrt[3]{\frac{12.I_0}{a_n}} = \sqrt[3]{\frac{12.2772}{201}} = 5,49\text{cm} \qquad (13)$$

where:
a_n is a distance between the radial girders in point of joint to the tank’s shell ;
t_{ek} – equivalent thickness of the roof's cover plates;
I_0 – common moment of inertia of the girder and roof sheet with thickness $\boldsymbol{a_n}$.

Here, in this research, contribution of roof's cover plates is not considered. Accepted in formula (13) value for I_0 = I_y = 2772 cm^4, as much as is the moment of inertia of the radial girder IPE 220.

To be valid the applied in Evolution Group for EN 1993-1-6 methodology, the following condition should be matched:

$$100 \leq \frac{R_r}{t_{ek}} \leq 3000 \qquad (14)$$

$\frac{R_r}{t_{ek}} = \frac{9600}{5{,}49} = 1748{,}6 < 3000$ → the shown statement could be used .

Critical value of the applied pressure, in elastic range, will be calculated according to:

$$p_{r,cr} = \frac{2}{\sqrt{3.(1-\nu^2)}}.C_c.E.\left(\frac{t}{R_r}\right)^2 = \tag{15}$$

$$\frac{2}{\sqrt{3.(1-0{,}3^2)}}.0{,}7.21000.\left(\frac{5{,}49}{9600}\right)^2 = 0{,}00582\ \mathrm{kN/cm^2}$$

where:
C_c = 0,7 is a coefficient, accounting conditions of shell's supports in its periphery;
E = 21 000 kN/cm^2 – module of the steel elasticity;
t = t_{ek} = 5,49 cm – equivalent thickness of the smooth spherical shell;
ν = 0,3 – coefficient of Poisson.

Critical value of the applied external pressure, in plastic range, will be calculated according to:

$$p_{r,pl} = f_{y,k}.C_{pl}.\frac{t}{R_r} = 35{,}5.0{,}9.\frac{5{,}49}{9600} = 0{,}0365\ \mathrm{kN/cm^2} \tag{16}$$

where:
$f_{y,k}$ = 35,5 kN/cm^2 is a characteristic value of yield strength of steel S355;
C_{pl} = 0,9 – coefficient, accounting supporting conditions of the shell.

The amplitude of characteristic imperfection Δw_k will be calculated as follow:

$$\Delta w_k = \frac{1}{Q}.\sqrt{R_r.t} = \frac{1}{16}.\sqrt{9600.5{,}49} = 14{,}348\ \mathrm{cm} \tag{17}$$

where:
Q is a parameter, accounting production quality. Q = 16 in case of normal quality.

The imperfection reduction factor α_I will be calculated according to:

$$\alpha_I = \frac{1}{1+1{,}90.\left(\frac{\Delta w_k}{t}\right)^{0{,}75}} = \frac{1}{1+1{,}90.\left(\frac{14{,}348}{5{,}49}\right)^{0{,}75}} = 0{,}2038 \tag{18}$$

Coefficient accounting the geometry imperfection α_G = 0,70.

The elastic imperfection factor α depends on directly by α_I and α_G, and can be calculated by formula:

$$\alpha = \alpha_I.\alpha_G = 0{,}2038.0{,}70 = 0{,}1427 \tag{19}$$

Relative slenderness $\bar{\lambda}$ will be calculated according to:

$$\bar{\lambda} = \sqrt{\frac{p_{r,pl}}{p_{r,cr}}} = \sqrt{\frac{0{,}0365}{0{,}00582}} = 2{,}504 > \lambda_0 = 0{,}20 \tag{20}$$

where:
λ_0 = 0,20 is squash limit relative slenderness.

Plastic limit relative slenderness of the shell $\bar{\lambda}_p$ will be calculated according to:

$$\bar{\lambda}_p = \sqrt{\frac{\alpha}{1-\beta}} = \sqrt{\frac{0{,}1427}{1-0{,}7}} = 0{,}6897 < \bar{\lambda} = 2{,}504 \tag{21}$$

where:
β = 0,70 is a plastic range factor in buckling interaction;
η = 1,0 - interaction exponent .

The loss of the stability coefficient χ will be calculated according to:

$$\chi = \frac{\alpha}{\bar{\lambda}^2} = \frac{0{,}1427}{2{,}504^2} = 0{,}0228 \tag{22}$$

In plastic range:

$$R_{pl} = \frac{p_{r,pl}}{p_{Ed}} = \frac{p_{r,pl}}{q_1} = \frac{0{,}0365.100^2}{1{,}935} = 188{,}63 \tag{23}$$

In elastic range:

$$R_{el} = \frac{p_{r,cr}}{p_{Ed}} = \frac{p_{r,cr}}{q_1} = \frac{0{,}00582.100^2}{1{,}935} = 30{,}08 \tag{24}$$

The coefficient R_k, indicates how many times we can increase the loads before that shell will lose stability, characteristically, will be calculated according to the formula:

$$R_k = \chi.R_{pl} = 0{,}0228.188{,}63 = 4{,}3 \tag{25}$$

Design value of the reserve of bearing capacity R_d, is determined below:

$$R_d = \frac{R_k}{\gamma_{M1}} = \frac{4{,}3}{1{,}05} = 4{,}095 > 1 \tag{26}$$

From the equation (26) it follows that the roof dome constructed by 100 pieces of radial girders IPE 220, considered as a equivalent smooth shell will not lose stability when the loads are symmetrically and evenly distributed.

- methodology of Вольмир (1956)

With sufficient correctness for practice, for spherical shells with ratio R_r / t = 400 ÷ 2000 and central vertical angle of the dome θ = 2.α_n = 40^0 ÷ 120^0 , critical values q_{cr} of external pressure, when shell will lose stability, can be determined. The formula is as follows:

$$q_{cr} = 0{,}3.k.E.\left(\frac{t_{ra}}{R_r}\right)^2 \tag{27}$$

where:

k is coefficient, calculated by formula:

$$k = \left(1 - 0{,}175.\frac{\theta^0 - 40^0}{40^0}\right).\left(1 - 0{,}07.\frac{R_r}{400.t_{ek}}\right) = \\ = \left(1 - 0{,}175.\frac{40^0 - 40^0}{40^0}\right).\left(1 - 0{,}07.\frac{9600}{400.5{,}49}\right) = 0{,}694 \tag{28}$$

$$\left.\begin{aligned} q_{cr} &= 0{,}3.k.E.\left(\frac{t_{ek}}{R_r}\right)^2 == 14{,}3\mathrm{kN/m}^2 \\ q_1 &= 1{,}935\mathrm{kN/m}^2 \end{aligned}\right\} \Rightarrow q_{cr} > q_1 \tag{29}$$

According to used methodology of Вольмир (1956), spherical shell should not loses stability loaded by load combination q_1.

- methodology, written in "Справочник проектировщика" (1973)

According to that methodology, the critical values q_{cr} of external pressure, wherein spherical shells will lose stability could, be determined by expression:

$$q_{cr} = \frac{E.t^4}{R^4}.\left[K_1.\frac{f}{t}\left(K_2.\frac{f^2}{t^2} + 1\right) + K_3.\sqrt{\left(K_4.\frac{f^2}{t^2} - 1\right)^3}\right] = \\ \frac{21000.5{,}49^4}{3200^4}.\left[1{,}52.\frac{549}{5{,}49}\left(0{,}00576.\frac{549^2}{5{,}49^2} + 1\right) + 0{,}407.\sqrt{\left(2{,}07.\frac{549^2}{5{,}49^2} - 1\right)^3}\right] = \\ = 0{,}22213\mathrm{kN/cm}^2 = 2221\mathrm{kN/m}^2$$

where:

t is thickness of the shell;

R - radius of circular base of spherical shell;

f - height of dome roof;

K_1, K_2, K_3, K_4 - coefficients depending on supporting conditions of dome roof.

Comparing values of load combination q_1 and critical values q_{cr} of external pressure, accounted by formula (30) is clear that shell should not lose stability. It should be noted that according to this methodology, calculated values for q_{cr} are unrealistically height.

6. Determining internal forces in radial girders with section IPE 220, when roof plates are loaded by combination $q_2 \uparrow$

A spatial computing model is created in order to determine internal forces in radial girders IPE 220 and their influence on thin steel plates. Software product SAP2000 v.14.2 is used. The model includes all elements of the roof - radial girders, cover plates, central ring and top angle, see Fig. 5.

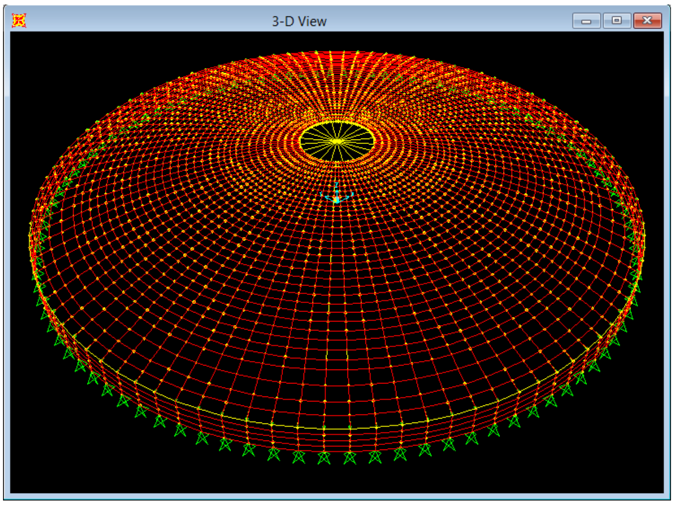

Fig. 5. *Model of dome roof, where cover plates are included*

Values of internal forces are accounted on some points along the radial girders. Strength check is made to the sufficiency of sections:

M_{max} = -18, 779 kN.m

N_c = + 63,391 kN

$$\sigma_{max} = \frac{63{,}391}{33{,}4} + \frac{18{,}779.100}{285} = \\ = 8{,}49\mathrm{kN/cm}^2 < \frac{f_y}{\gamma_{M0}} = \frac{35{,}5}{1{,}05} = 33{,}81\mathrm{kN/cm}^2 \tag{31}$$

N_{max} = + 98,64 kN

M_c = - 13,438 kN.m

$$\sigma_{max} = \frac{98{,}64}{33{,}4} + \frac{13{,}438.100}{285} = \\ = 7{,}67\mathrm{kN/cm}^2 < \frac{f_y}{\gamma_{M0}} = \frac{35{,}5}{1{,}05} = 33{,}81\mathrm{kN/cm}^2 \tag{32}$$

By analysis of the stresses in roof sheets of load combination $q_2 \uparrow$, see Fig. 6, it is seen that presence of radial girders IPE 220 on practice does not change membrane stress of roof plates. In other words, has a reason to use Laplace's equation, shown on formula (8).

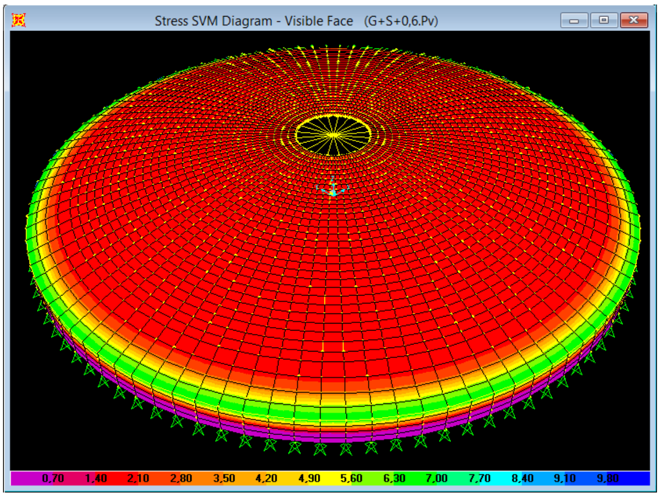

Fig. 6. *Normal stresses by von Mises in roof cover plates, kN/cm*^{2}

7. Conclusions

Self-supporting steel dome roofs are built by girders and cover plates. When these elements are reliable connected, dome roofs could be considered as a spatial smooth shells. And apply to them methods and analyzes of number of researches working on shells. As a result, taking into account spatial work of plates and structural elements, can significantly reduce amount of steel.

Basic disadvantage of used there approach is that all researches, respectively normative documents, consider only cases of a uniformly distributed loads. A snow on the roof, blowing by wind, can be moved, to accumulate, wherein the load becomes asymmetrical, i.e., less favourable.

On other hand, European standard for snow loads EN 1991-1-3:2006 do not considers case of an uneven load on spherical domes.

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Lyubomir ZDRAVKOV – PhD, Associate Professor, Civil Engineer, Department “Steel and Timber structures”

Main research area: steel structures

Address: University of Architecture, Civil Engineering and Geodesy, Faculty of Civil Engineering, №1 “Hristo Smirnensky” str., floor 7, Sofia 1046, Bulgaria

e-mail: zdravkov_fce@uacg.bg