NECESSARY HEIGHT OF THE VERTICAL STIFFENERS IN STEEL SILOS ON DISCRETE SUPPORTS WITH ACCOUNTING OF IMPERFECTIONS

Lyubomir A. ZDRAVKOV

PhD, associate professor, civil engineer
Faculty of Structural Engineering, UACEG
Sofia 1046, №1 „Hristo Smirnensky" str., floor 7, office 733, Bulgaria
e-mail: zdravkov_fce@uacg.bg

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ABSTRACT

To ensure unloading of the whole amount of stored product by gravity, the steel silos are often placed on supporting frame structure. The values of stresses in the joints between the thin shell and supporting frame elements are extremely high. It can cause local loss of stability in the shell. To prevent it, many designers place stiffening elements above the supports. Here the question is how high should be the stiffening elements? The appropriate solution is that they should rise to that level till which the values of meridional normal stresses above the supports and in the middle between them are equalized. But where is this level? Many researchers worked on values and ways of distribution of normal meridional stresses above the supports of the cylindrical shells. As a result of their efforts are determined critical height H_{cr} *of the shell and the ideal position* H_I *of intermediate stiffening ring. But these heights are considerably different between each other. To which of them our vertical stiffening elements should achieve? Considering the nonlinear behaviour of the steel, the effects of changes in geometry during loading and imperfections caused by welding works, the author tried to obtain an answer to this question.*

Keywords: *steel silos, meridional stress, critical height, vertical stiffener, loss of stability, material and geometrical nonlinearity, imperfection*

1. INTRODUCTION

Typically steel silos are elevated facilities, placed on supporting structure. The purpose is to unload all stored product easily and completely by gravity. Steel structure beneath the silos is different for every project, depending on real conditions of exploitation. The most common are two types – built by horizontal girders and columns or by vertical columns only. Both types of frame structures cause a concentration of meridional forces in the cylindrical body of the silos. As a result, the thin-walled shell could lose local stability.

The simplest way to design steel silos is to divide in our minds the thin cylindrical shell into two parts - discretely supported ring beam and continuously supported shell above it. This conception is accepted by the European standard EN 1993-4-1 [7], see Fig. 1. To ensure continuous support of the shell, the bending stiffness of the ring beam should be high. Unfortunately EN 1993-4-1 keeps silence about recommended stiffness of ring beam.

Rotter [10] suggested in 1985 that a value of the ratio between the stiffness of cylindrical shell and ring beam $\psi = 0.25$ might be suitable for adoption in design, where:

$$\psi = \frac{K_{shell}}{K_{ring}} , \qquad (1)$$

in which:
K_{shell} is the stiffness of cylindrical shell;
K_{ring} - stiffness of ring beam.

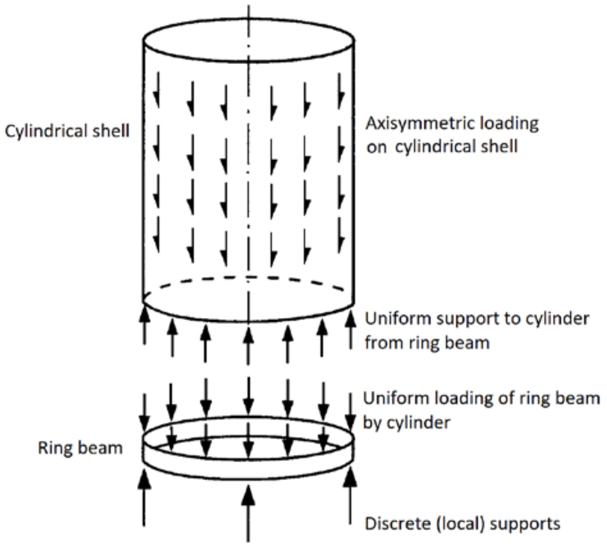


***Figure 1:** Traditional design model for silos on discrete supports*

Based on the English translation of study of *Vlasov* [15] about curved beams, stiffness of ring beam K_{ring} is expressed as:

$$K_{\text{ring}} = \frac{(n^2-1)^2 EI_{\text{r}}}{R^4} \cdot \frac{1}{f_{\text{r}}} , \tag{2}$$

where:

n is the number of uniformly spaced supports;

E - modulus of elasticity;

I_{r} – the moment of inertia about a radial axis;

R – the radius of ring beam centroid.

$$f_{\text{r}} = 1 + \frac{EI_{\text{r}}}{n^2 K_{\text{T}}} , \tag{3}$$

in which:

$$K_{\text{T}} = GJ + n^2 \frac{EC_{\text{w}}}{R^2} , \tag{4}$$

where:

G is shear modulus;

J - torsional constant;

C_{w} - warping constant for an open section.

Semimebrane theory of shells, proposed by *Vlasov* [16], gives an expression of the stiffness of cylindrical shell, as follow:

$$K_{\text{shell}} = n\sqrt{(n^2-1)}\frac{E}{\sqrt[4]{3}}\left(\frac{t}{R}\right)^{3/2} \cdot \frac{1}{f_{\text{s}}} , \tag{5}$$

where t is a thickness of the cylindrical shell.

$$f_{\text{s}} = \frac{(e^{\eta})^2 - 2 \cdot e^{\eta} \cdot \sin(\eta) - 1}{(e^{\eta})^2 - 2 \cdot e^{\eta} \cdot \cos(\eta) + 1} , \tag{6}$$

in which:

$$\eta = \frac{2\pi H}{\mu} , \tag{7}$$

where:

H is the height of the cylindrical shell;

μ – expressed by *Calladine* [3] long wave bending half-wavelength:

$$\mu = \frac{2\pi\sqrt[4]{3}}{n\sqrt{(n^2-1)}}\sqrt{\frac{R}{t}}R \tag{8}$$

Based on equations (2) and (5), stiffness ratio ψ will look like:

$$\psi = \frac{K_{\text{shell}}}{K_{\text{ring}}} = \frac{0.76(Rt)^2}{I_{\text{r}}}\sqrt{\frac{R}{t}}\sqrt{\frac{n^2}{(n^2-1)^3}} \cdot \frac{f_{\text{r}}}{f_{\text{s}}} \tag{9}$$

For simplification, the equation (6) could be represented by two ordinary relations:

$$f_{\text{s}} = \begin{cases} \frac{\eta}{3}, \text{ when } H \le H_{\text{cr}} \\ 1.0, \text{ when } H > H_{\text{cr}} \end{cases} \tag{10}$$

where H_{cr} is the critical height of the cylindrical shell. It could be calculated according to (17).

Later, *Topkaya* and *Rotter* [11,12] conducted extensive finite element analyses for verification of Rotter's criterion about the necessary stiffness of the ring beam. With 1280 separate finite-element analyses (FEA), including two different types of ring sections, various heights and radii of cylindrical shells, the researchers examined the validity of suggested by *Rotter* [10] ratio $\psi = 0.25$. Based on done FEA *Topkaya* and *Rotter* concluded, when a stiffness ratio $\psi \le 0.1$, axial stresses will not deviate more than 25% from the uniform support assumption.

***Figure 2:** Stiffening elements above discrete supports of the shell*

It should be noted that all above-mentioned researches are conducted with smooth steel shells, without vertical stiffeners in them. On the other side, a common practice in the design of steel structures is to place stiffening elements on the point, where are applied concentrated loads. In our case, the stiffeners should be placed above the discrete supports, see Fig. 2.

In his research *Zdravkov* [19] shows that when there are vertical stiffening elements without stiffening ring on the upper end, the height of the critical zone, in which are distributed vertical reactions of discrete supports, is increased. In other words as longer are the stiffeners, as higher will be the critical zone. Then the question is how long have to be the stiffeners? The appropriate solution is they should rise to that level till which the values of the meridional normal stresses above the supports and in the middle between them are equalized. Beneath this level the cylindrical shell will be considered as a ring beam, stiffened by elements above the supports. Above it, the cylinder could be calculated as a continuously supported shell. But where is this level? In another research *Zdravkov* [22] tried to resolve this question. In that article, he recommends the limits of the height H_S of stiffeners to be $H_I \leq H_S \leq H_L$, where H_I is the ideal height for intermediate stiffening ring, calculated according to the formula (14). H_L is a distance between discrete (local) supports, calculated through equation (16). The research of *Zdravkov* [22] is carried out with ideal cylindrical shells considering the geometrical nonlinearity (GNA). On another hand, the thin-walled shells are sensitive for imperfection in them. Should not forget also the real structural steel has yield strength i.e. the relation $\sigma - \varepsilon$ is not linear. Is it possible material nonlinearity and geometrical imperfections in the thin-walled shells to have an influence on the necessary height H_S of the vertical stiffeners? In this research, the author will try to determine if it is accurate.

2. ANALYSIS

For the current research, four steel cylindrical shells are modelled, using commercial software ANSYS. Their parameters are as follow:

a) dimensions:
- shell 1 – diameter D = 2 m, height H = 4 m;
- shell 2 – diameter D = 3 m, height H = 6 m;
- shell 3 – diameter D = 4 m, height H = 8 m;
- shell 4 – diameter D = 5 m, height H = 10m.

b) all shells are with constant thickness t = 5 mm;

c) the stored in the facilities products are:
- shell 1 – flour;
- shell 2 – cement;
- shell 3 – slacked lime;
- shell 4 – sand.

Every product causes horizontal pressure P_{hf} and vertical P_{wf} load due to the friction between the stored material and the shell, see Fig. 5. Their values are determined for every particular product according to standard EN 1991-4 [5]. All loads are uniformly distributed and applied as a surface pressure on the shell. They are applied to the internal surface of the shells.

d) used steels in the models are as follow:
S235 – for shells 1, 2 and 3;
S355 – for shell 4.

The mechanical characteristics of steels S235 and S355 are according to standard EN 10025-2:2004 [8], namely:

- yield strength f_y:
f_y = 235 MPa in S235 and f_y = 355 MPa in S355.

- modulus of elasticity – E = 210 000 MPa;
- coefficient of *Poisson* - ν = 0,3.

e) material nonlinearity of the steel is considered in the research (MNA). The relation between the stresses σ and strains ε is bilinear. Idealized diagram of *Prandtl* is used, see Fig. 3;

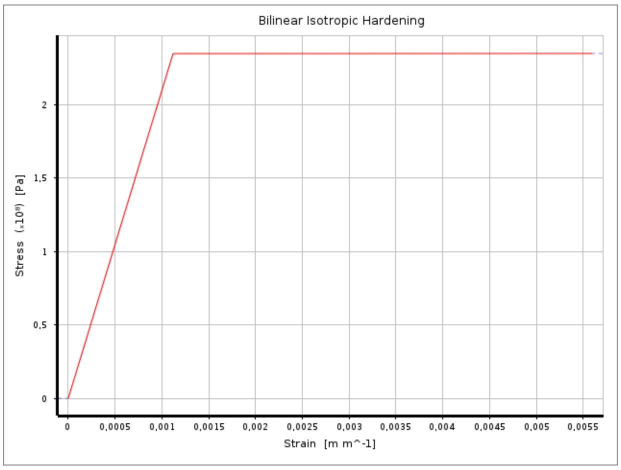


Figure 3: *Idealized diagram stress-strain*

f) the thin-walled shells are sensitive to the effects of change of the geometry during loading. On that reason all models are researched considering the geometrical nonlinearity (GNA);

g) shells 1 ÷ 4 are researched when:

- cylindrical shells are perfect, with an accounting of material and geometrical nonlinearity (GMNA), described in the standard EN 1993-1-6 [6];

- cylindrical shells have initial geometrical imperfections, see Fig. 4, with an accounting of material and geometrical nonlinearity (GMNIA).

h) the imperfections are symmetrically positioned toward the vertical axis on the entire circumference of the shells. They simulate radial deformations due to welding operations in horizontal joints. Their shape is shown in Fig. 4.

Figure 4: *Geometrical imperfections of the shell. Dimensions*

i) shown on Fig. 4 length l_{wg} of the imperfection is calculated according to the standard EN 1993-1-6 [6], by the formula:

$$l_{gw} = 25 \cdot t = 25 \cdot 5 = 125 \text{ mm} \quad (11)$$

Like the researches of *Doerich* and *Rotter* [4], *Vanlaere et al.* [14], the author accepted that the radial deflection $\Delta\omega_{0w} = t = 5$ mm. It is bigger than the calculated by EN 1993-1-6:

$$\Delta\omega_{0w} = U_{0w} \cdot l_{gw} = 0.016 \cdot 125 = 2 \text{ mm}, \quad (12)$$

where $U_{0w} = 0.016$ is the dimple tolerance parameter for the fabrication tolerance quality class C.

In this way, the author wanted to underline the effect of the geometrical imperfections.

j) cylindrical shells are made by courses with height of 2 000 mm. It means that initial imperfections will be several and will be positioned at the distance 2 000 mm by height;

k) all shells are supported by six immovable supports with dimensions in plane 125×125 mm, see Fig. 5.

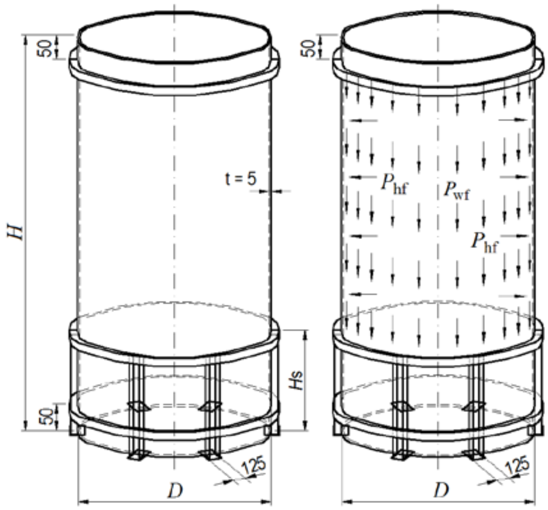


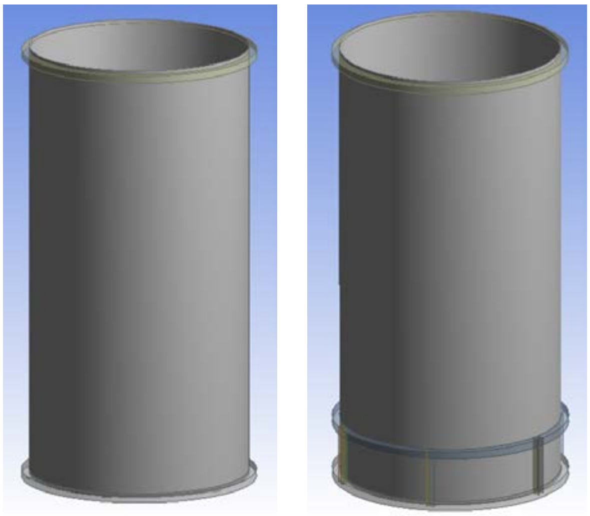
Figure 5: *Numerical models – dimensions and loading*

l) to strengthen the shells in a radial direction, on 50mm above the lower edge and 50mm below the upper edge are placed rings with a section L100x8 mm, welded as is shown in Fig. 8;

m) shells 1 ÷ 4 are researched in six different types of stiffening on the supports:

- models without stiffeners above the supports, see Fig. 6*a*;

- the shells are stiffened with an intermediate ring with section L100x8 mm, see Fig. 6*b*. Above every support, outside of the shell, are placed 2 steel plates with a section 8x100 mm, which reach to the intermediate ring.



a) shell without vertical stiffeners *b) shell with vertical stiffeners*

Figure 6: *Vertical stiffening elements on the cylindrical shell*

The levels H_s, reached by vertical stiffeners, see Fig.4, are determined as follows:

- using an average value of the distribution of discrete forces F_R from supports α = 45°, see Fig. 7. The height H_{45} is determined with the expression:

$$H_{45} = \frac{\pi R}{n} \tag{13}$$

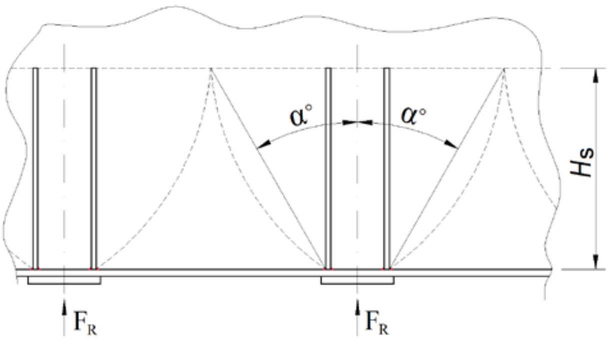


Figure 7: *Average angle α of distribution of the compressive forces on height*

- at ideal position of the intermediate stiffening ring on the shell.

Topkaya and *Rotter* [13] determined the ideal location of intermediate stiffening rings on the shell. They expect a ring, placed at this ideal position, can effectively remove all circumferential nonuniformity in the axial membrane stress above it. The simple expression of ideal location H_I is:

$$H_I = \sqrt{12(1+\nu)}\frac{R}{n}, \tag{14}$$

where ν is the coefficient of *Poisson*.

- using an average value of the distribution of discrete forces F_R from supports α = 30°. The height H_{30} should be calculated by the formula:

$$H_{30} = \frac{\pi R}{n}\mathrm{tg}(90^\circ - \alpha^\circ) \tag{15}$$

- the length of the stiffeners H_L is equal to the distance between the supports. It is calculated according to the formula:

$$H_L = \frac{2\pi R}{n} \tag{16}$$

- the length of the stiffeners is equal to the critical height in the shell H_{cr}.

H_{cr} represents the effective shell's height which is effective in redistributing of discrete forces from supports and equalizing of axial normal stresses. When the height of shell $H \le H_{cr}$, the entire shell resists axial loads from supports. When $H > H_{cr}$, only that part between the bottom of the shell and critical height H_{cr} is effective in redistributing of vertical reactions from discrete columns. H_{cr} can be calculated through the expression:

$$H_{cr} = \frac{3\sqrt[4]{3}}{n\sqrt{(n^2-1)}}\sqrt{\frac{R}{t}}\cdot R \tag{17}$$

The angular section L100x8 and a part of the cylindrical shell form an intermediate stiffening ring with a shape as is shown in Fig. 8.

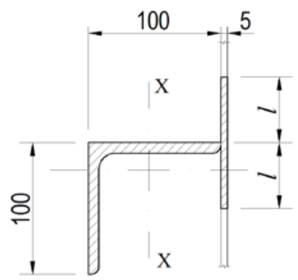


Figure 8: *Shape of the intermediate stiffening ring*

Effective width l of the steel sheets over and below the joint is calculated according to the standard API 650, by the expression:

$$l \le 13.4\cdot\sqrt{Dt}, \tag{18}$$

where:

D is the diameter of a cylindrical shell, m;

t – thickness of the cylindrical shell, mm.

Effective width l for the shells with the smallest diameter, D = 2 m, is equal to:

$$l \le 13.4\cdot\sqrt{Dt} = 13.4\cdot\sqrt{2\cdot 5} = 42.4mm.$$

The author is accepted to have an effective width l = 42 mm for all shells. It is on a way of safety.

The geometrical parameters of the formed stiffening rings are:
a) area - A = 20.11 cm^2 ;
b) moment of inertia about vertical axis „x-x" - I_x=345 cm^4.

Necessary stiffness of intermediate stiffening rings is determined by *Zeybek et al*. [17]. Stiffness ratio χ could be expressed as:

$$\chi = \frac{K_{shell}}{K_{stiffener}} = \frac{Rt\left(AR^2 + I_x.n^2(n^2-1)\right)}{12\sqrt{3}(1+\nu)^{1,5}\cdot AI_x n(n^2-1)^2}, \tag{19}$$

where:
K_{shell} is circumferential stiffness of the shell;
$K_{stiffener}$ - circumferential stiffness of circular ring;
A – cross-sectional area of the stiffening ring;
I_x - moment of inertia of the stiffening ring about vertical axis "x-x".

The results in research of *Zeybek et al.* [17] indicate that ratios below about $\chi < 0.2$ provide a satisfactorily uniform axial membrane stress distribution above the intermediate ring stiffener, so this limit is recommended for practical design. In his later research *Zeybek et al.* [18] confirmed, that correlation smaller than $\chi < 0.2$ are sufficient even when the rings are placed under their ideal position.

For different shells, the ratio of the stiffness's χ, calculated according to the formula (19), received values as follow:

- shell 1 – χ =0.0202;
- shell 2 – χ = 0.042;
- shell 3 – χ = 0.078;
- shell 4 – χ = 0.135.

The maximum value of the ratio $\chi = 0.130 < 0.2$, so it could be expected that the stiffness of the intermediate ring will be sufficient to equalize the meridional stresses in the shell above it.

All shells are modelled by 2D element shell281. The method of its creation is "Quadrilaterals". The finite elements are quadrilateral with eight joints - in the edges and the middle of the side. The maximum dimension of the elements is 50 mm.

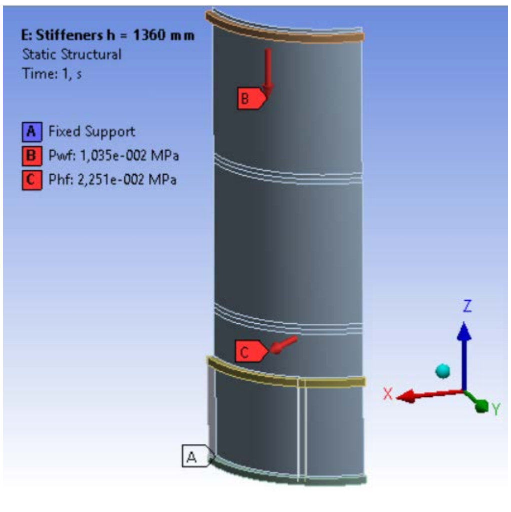


***Figure 9:** A quarter of the silos, used in the numerical analysis*

Option "symmetry" is activated in ANSYS. The purpose is to decrease the necessary time for calculation. In analysis is used a quarter of silo only, see Fig. 9.

Axial normal stresses are accounted by the height of shell, in the middle between two supports and above the supports. After that are determined the values of ratio $\sigma_{x,m}/\sigma_{x,s}$, where:

$\sigma_{x,m}$ is meridional normal stress by the height of the cylinder, in the middle between two supports;

$\sigma_{x,s}$ – meridional normal stress by the height, above the supports.

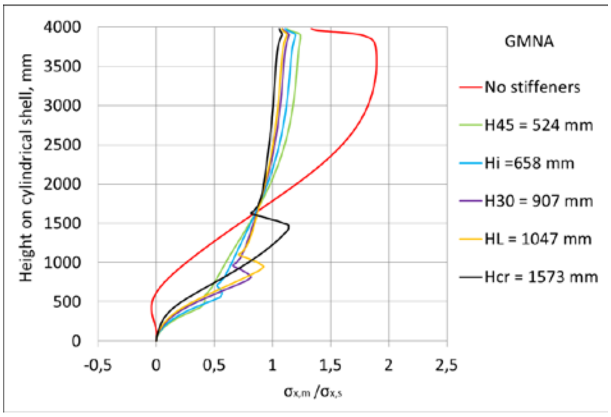
The idea is that where the ratio $\sigma_{x,m}/\sigma_{x,s}$ =1.0, is the upper border of the critical zone in the shell, in which are redistributed vertical reactions of supports.

3. RESULTS

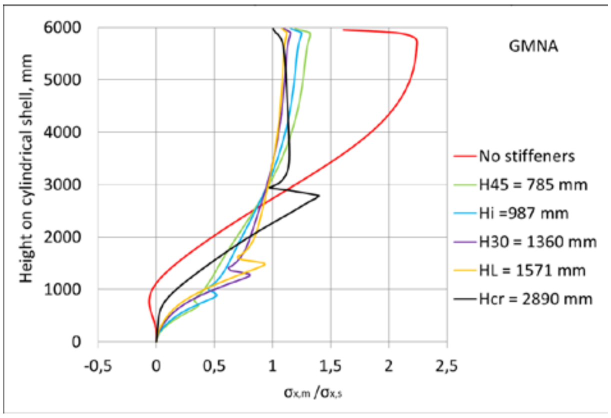
In Fig. 1 could be seen the change of the ratio $\sigma_{x,m}/\sigma_{x,s}$ by the height of the shell when GMNA is executed, and in Fig. 11 – during GMNIA. The author does not identify considerable differences in the shape of the graphics, excluding zones around the geometrical imperfections. And in these zones, the difference is up to 10%. The conclusion is that the geometrical imperfections as above described welding deformations do not reshape the global picture of change by the height of the ratio $\sigma_{x,m}/\sigma_{x,s}$.

In Fig. 10 and Fig. 11 could be seen ratios $\sigma_{x,m}/\sigma_{x,s}$ > 1.0. It means that in part of the shell the axial stresses in the middle between the supports are bigger than the stresses above the supports. A similar phenomenon has been observed in the researches of *Knödel* and *Ummenhofer* [9], and *Zdravkov* [19,20]. This effect is underlined in shells without intermediate rings, in which could be reported values of the ratio $\sigma_{x,m}/\sigma_{x,s}$ > 2.0.

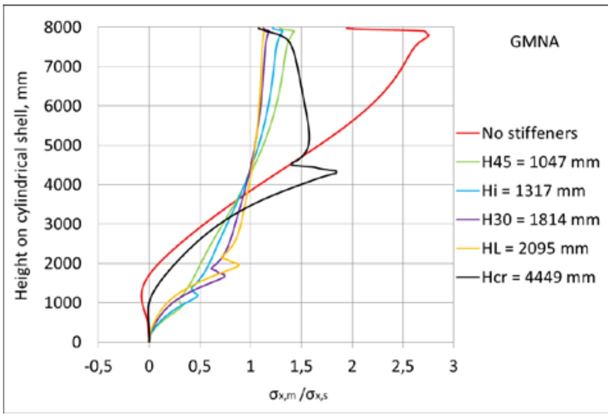
It could be seen that as longer are the vertical stiffening elements as bigger are the ratios $\sigma_{x,m}/\sigma_{x,s}$ above their upper end, i.e. the inequality of the stresses above them is smaller. Exceptions over here are the stiffening elements with height $H_s = H_{cr}$.



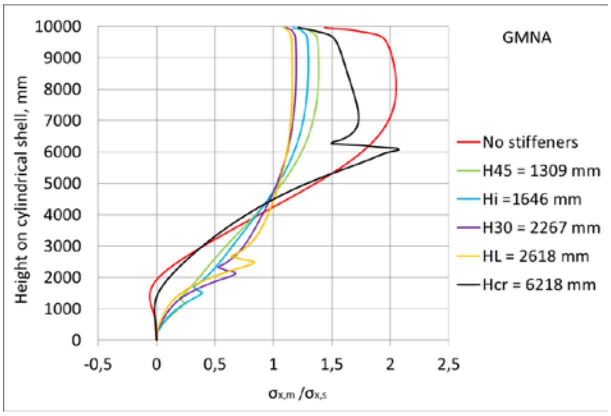
a) shell 1 – diameter D = 2 m, height H = 4 m



b) shell 2 – diameter D = 3 m, height H = 6 m

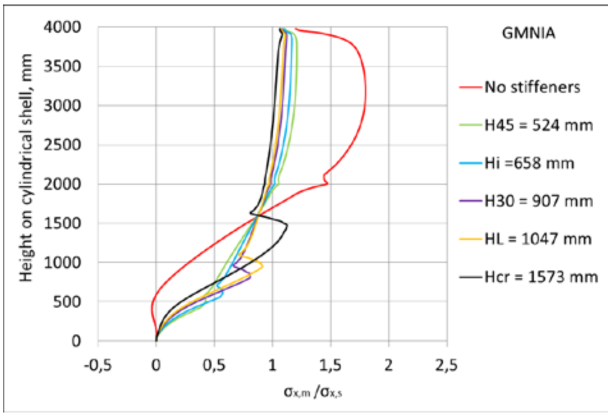


c) shell 3 – diameter D = 4 m, height H = 8 m

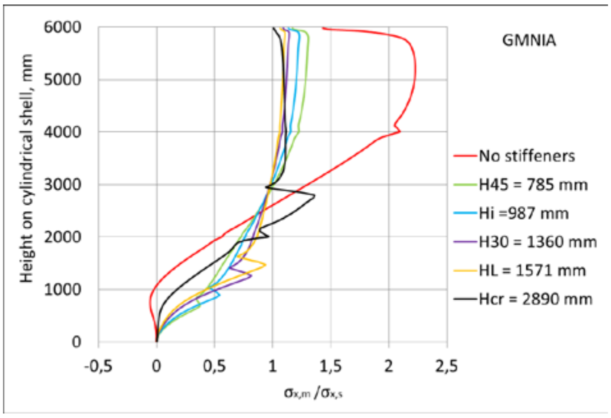


d) shell 4 – diameter D = 5 m, height H = 10 m

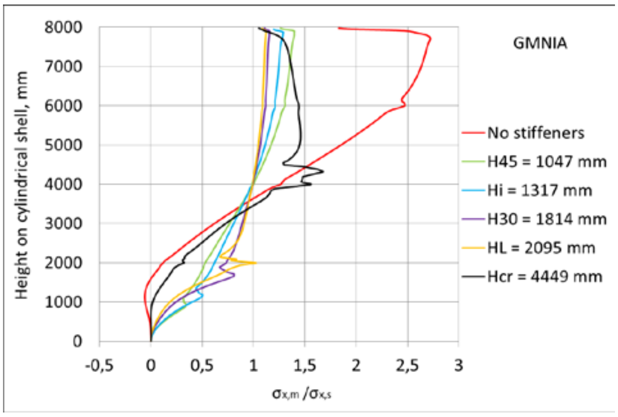
***Figure 10**: Change in the ratio $\sigma_{x,m}/\sigma_{x,s}$ by the height of the ideal cylindrical shell*



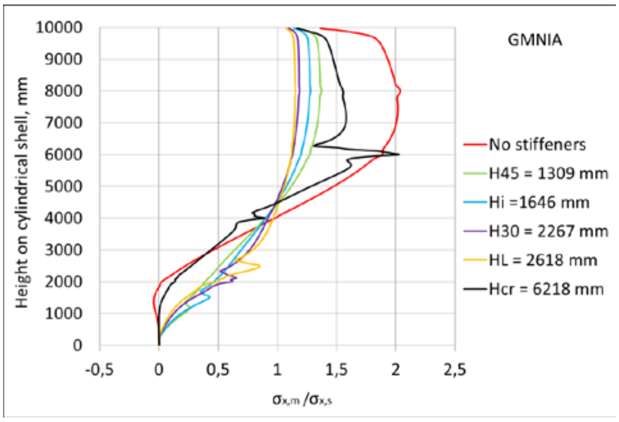
a) shell 1 – diameter D = 2 m, height H = 4 m



b) shell 2 – diameter D = 3 m, height H = 6 m



c) shell 3 – diameter D = 4 m, height H = 8 m



d) shell 4 – diameter D = 5 m, height H = 10 m

Figure 11: *Change in the ratio $\sigma_{x,m}/\sigma_{x,s}$ by the height of the imperfect cylindrical shell*

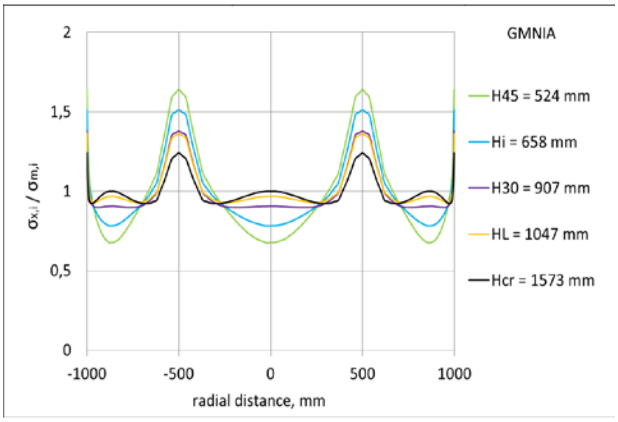
It is obvious that the use of vertical stiffening elements considerably increases the bearing capacity of the shell on meridional loading. Longer stiffeners assure a bigger reserve of bearing capacity. The effect is not linear and decreases with elongation of the stiffeners.

As an additional tool for an evaluation of the inequality of meridional stresses, the author has accounted their values on level 50 mm above the intermediate stiffening rings. In Fig. 12 could be seen the change of the ratio $\sigma_{x,i}/\sigma_{m,i}$ on this level, where:

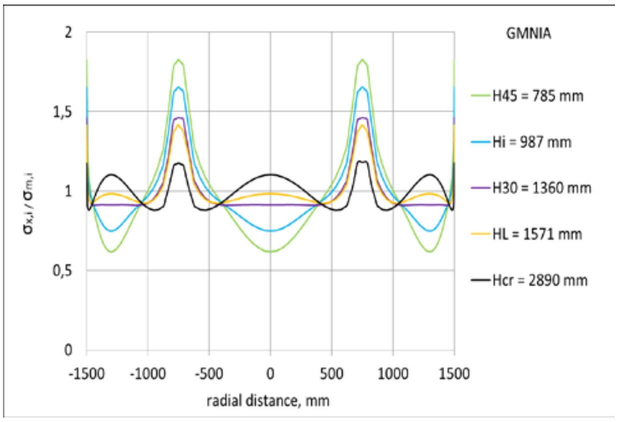
$\sigma_{x,i}$ is the real meridional normal stress on level 50 mm above the intermediate stiffening ring;
$\sigma_{m,i}$ – the mean meridional normal stress on the same level.

The graphics in Fig. 12 show that no exists equalizing of stresses above the intermediate stiffening rings, however high they may be positioned. On the other side, elongation of vertical stiffeners leads on to increase of meridional normal stresses $\sigma_{x,m}$ in the middle between two supports.

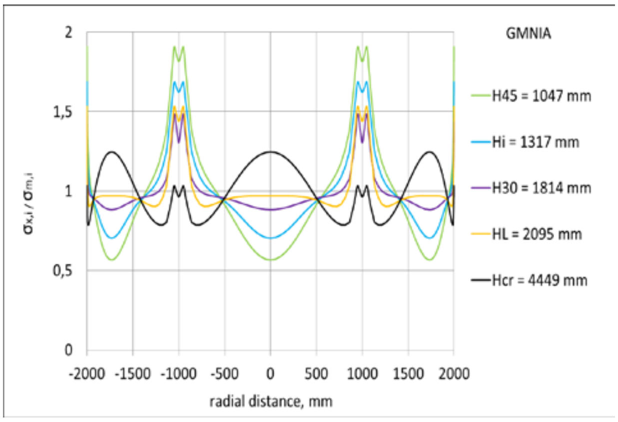
In his researches *Zdravkov* [19,20,21] shows that equalizing of the axial normal stresses on the height depends on many factors. As a presence, by vertical stiffeners and/or intermediate rings, the value of internal pressure, ratio *D*/*t* and other. For that reason, the author cannot state exactly how long should be the stiffening elements. He only can recommend using vertical stiffening with length $H_{30} \leq H_S \leq H_L$.



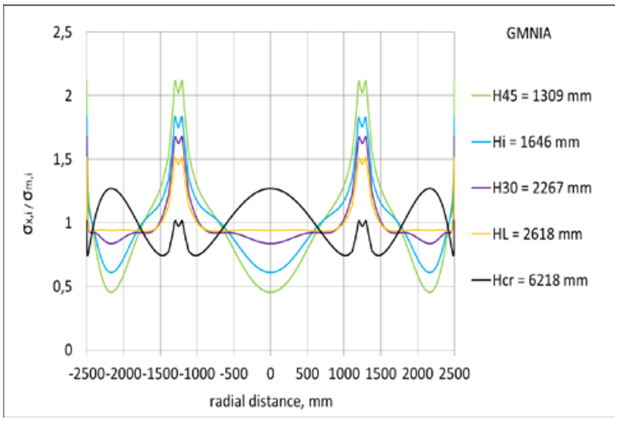
a) shell 1 – diameter D = 2 m, height H = 4 m



b) shell 2 – diameter D = 3 m, height H = 6 m



c) shell 3 – diameter D = 4 m, height H = 8 m



d) shell 4 – diameter D = 5 m, height H = 10 m

Figure 12: *Change in the ratio $\sigma_{x,i}/\sigma_{m,i}$ by level 50 mm above the intermediate stiffening rings*

4. CONCLUSION

A common practice in the design of steel silos is to place stiffening elements on applying points of concentrated loads. In our case, the stiffeners are positioned above the discrete supports. As a result, bearing capacity of the shell on meridional loads increases. Longer stiffeners assure bigger bearing capacity, but the effect decreases with elongation of stiffeners. The equalization of the axial normal stresses depends on many factors. Furthermore, the current GMNA and GMNIA research is carried with few models. Because of this, the author can only recommend the limits of the length H_S of the stiffeners. According to the author, it should be $H_{30} \leq H_S \leq H_L$. When the heights are $H_S \leq H_{30}$ the ratio $\sigma_{x,m}/\sigma_{x,s} < 0.75$, i.e. there still is a large inequality in the meridional normal stresses above the upper end of the stiffeners. There is no much sense to use stiffeners with a length $H_S > H_L$, because the effect of their elongation above these values is very small. Besides of it, the longer stiffening elements lead to ratio $\sigma_{x,m}/\sigma_{x,s} > 1.0$ in their upper end.

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